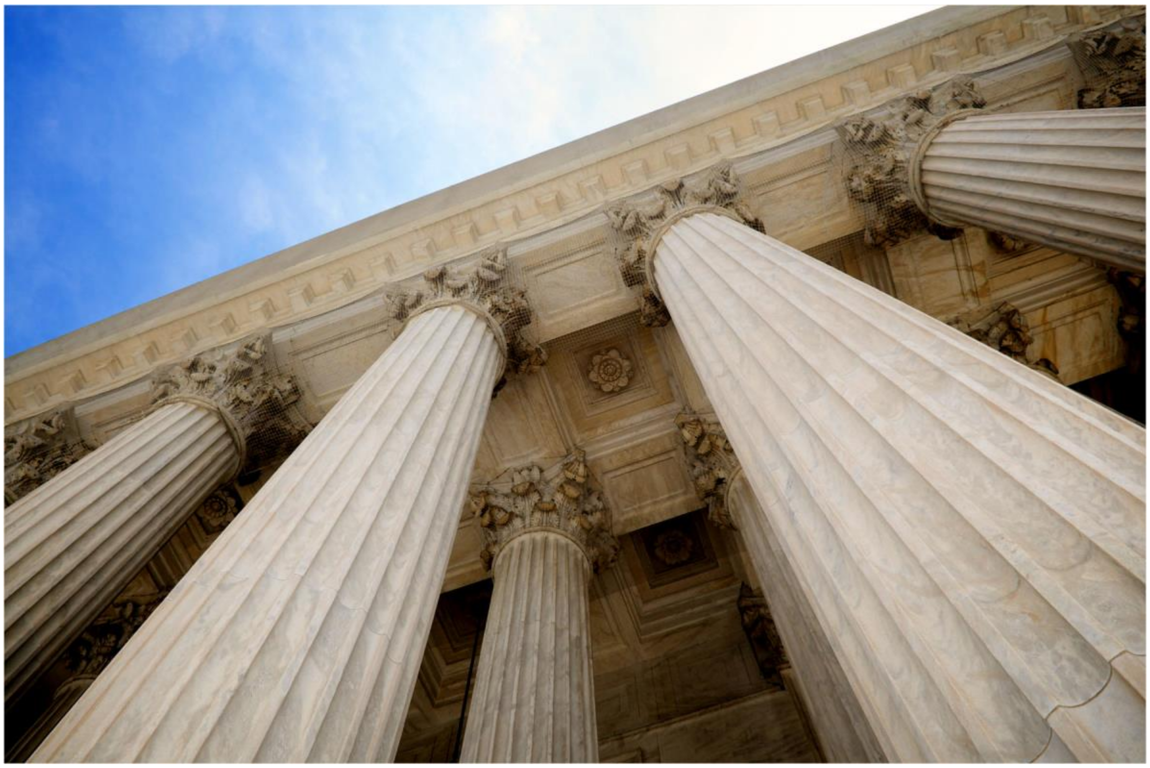

Functions in Mathematics

PROFESSIONAL MATHEMATICS

Sercan Külcü | Professional Mathematics | 10.01.2022

Contents

Chapter 1: Introduction to Functions in Mathematics

Welcome, students! Today we're going to be learning about a fundamental concept in mathematics: functions.

A function is a rule that assigns to each element in a set, called the domain, exactly one element in another set, called the codomain. We can think of a function as a machine that takes in an input (from the domain) and gives out an output (from the codomain). For example, we can define a function that takes in a real number as an input, and gives out its square as the output.

One way we can represent a function is by using function notation. We use the name of the function (often a letter like f or g) to represent the function as a whole, and we put the input inside parentheses, like this: $f(x)$. The input x is called the argument of the function. The output is then represented by the value of the function at the argument x, or $f(x)$.

An example of this is $y = f(x) = x^2$, where x = input, y = output and f = function.

A graph of a function is a visual representation of how the input and output are related. We can plot the points $(x, f(x))$ on a coordinate plane, and this will give us a graph of the function. For example, the graph of the function $f(x) = x^2$ is a parabola that opens upwards.

There are different types of functions, including linear functions, quadratic functions, and polynomial functions. We'll be exploring each of these types of functions in more detail in future lessons.

One important concept related to functions is the inverse function. If a function $f(x)$ assigns each element x in the domain a unique element y in the codomain, we can create a new function $g(x)$, called the inverse of f, such that $g(f(x)) = x$ for all x in the domain of f. In other words, the inverse function "undoes" the original function.

This is just a brief introduction to functions in mathematics. There's much more to explore and discover, such as domain, range, one-to-one and many-to-one functions, compositions and inverse functions, and much more. We'll be going over all of these concepts in more detail in the next chapter.

Thank you for joining me today. I hope you found this introduction to functions helpful and I can't wait for our next lesson!

Chapter 2: Types of Functions

Welcome back, students! In the last chapter, we learned about the basic concept of a function and how it can be represented using function notation. In this chapter, we're going to be exploring different types of functions and their characteristics.

The first type of function we'll be discussing is the linear function. A linear function is a function of the form $f(x) = mx + b$, where m and b are constants. The graph of a linear function is a straight line. Linear functions are important in many areas of mathematics, such as geometry and physics.

Next, we have quadratic functions. A quadratic function is a function of the form $f(x) = ax^2 + bx + c$, where a, b, and c are constants. The graph of a quadratic function is a parabola. The value of a determines whether the parabola opens upwards or downwards, and the value of b determines the direction of the vertex of the parabola.

A polynomial function is a function that can be written in the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$, where $a_n, a_{n-1}, \dots, a_1, a_0$ are constants and n is a non-negative integer. A polynomial function can have any degree, and the graph of a polynomial function of degree n will have at most n-1 turning points.

Another important type of function is the rational function. A rational function is a function of the form $f(x) = (p(x)) / (q(x))$, where p(x) and q(x) are polynomial functions. The graph of a rational function is the set of all points in the coordinate plane where the function is defined, with the exception of certain "holes" where the denominator q(x) is zero.

In addition to these types of functions, there are also logarithmic functions, exponential functions, and trigonometric functions, each with their own unique characteristics and applications. We'll be discussing these types of functions in future chapters.

In this chapter, we've learned about different types of functions, such as linear, quadratic, polynomial, and rational functions. We've also seen that each type of function has its own unique characteristics and applications.

As you continue to learn more about functions, it is important to practice graphing and understanding the behavior of the functions and the relations to their equations. Next chapter we will explore some of the properties and transformations of functions.

Thank you for joining me today. I hope you found this chapter on types of functions to be helpful and informative. I can't wait for our next lesson!

Chapter 3: Properties and Transformations of Functions

Welcome back, students! In the previous chapter, we learned about different types of functions and their characteristics. In this chapter, we'll be exploring some of the properties and transformations of functions.

One important property of a function is its domain. The domain of a function is the set of all input values (or x-values) for which the function is defined. For example, the domain of the function $f(x) = 1/x$ is all real numbers except 0, because dividing by zero is undefined.

Another important property of a function is its range. The range of a function is the set of all output values (or y-values) that the function can produce. For example, the range of the function $f(x) = x^2$ is all non-negative real numbers, because squaring any real number will always produce a non-negative result.

We can also perform transformations on a function to change its appearance on the coordinate plane. One common transformation is a shift. A shift occurs when we add or subtract a constant from the input or output of a function. For example, $f(x) = x^2$ is the parent function for a family of parabolas. We can shift this parabola upward, downward, right, or left by adding or subtracting a constant to the input or output.

Another common transformation is stretching or shrinking. A stretch or shrink occurs when we multiply or divide the input or output by a constant. For example, $f(x) = 2x^2$ is a stretch of the parent function $f(x) = x^2$ by a factor of 2 in the y-direction.

We also can reflect the graph of a function across the x- or y-axis by changing the sign of the input or output. For example, $f(x) = -x^2$ will reflect the graph of $f(x) = x^2$ across the x-axis.

These are just a few examples of the properties and transformations that we can apply to functions. Understanding these properties and transformations can be useful for graphing and analyzing functions, as well as for solving problems in various fields such as physics, engineering, and economics.

In this chapter, we've learned about the domain, range and various transformations that can be applied to a function, including shifts, stretches, shrinks, and reflections. We've also seen that understanding these properties and transformations can be useful in graphing and analyzing functions.

Next chapter we will be focusing on how to analyze functions, such as finding the maxima and minima, determining the intervals of increase and decrease and the concavity of the function.

Thank you for joining me today. I hope you found this chapter on properties and transformations of functions to be helpful and informative. I can't wait for our next lesson!

Chapter 4: Analyzing Functions

Welcome back, students! In the previous chapter, we learned about properties and transformations of functions. In this chapter, we'll be focusing on how to analyze functions, such as finding the maxima and minima, determining the intervals of increase and decrease, and understanding the concavity of the function.

One important aspect of analyzing a function is finding its critical points. A critical point is a value of x where the function has a local extreme value (either a maximum or minimum). To find the critical points, we must find the values of x where the derivative of the function is zero or undefined.

Another important aspect is determining the intervals of increase and decrease of a function. To do this, we must find the intervals of the domain where the derivative of the function is positive or negative, respectively.

Another way to understand the behavior of the function is to examine the concavity of the function. The concavity of the function is determined by the sign of the second derivative. If the second derivative is positive, the function is concave up. If the second derivative is negative, the function is concave down. At points where the second derivative is zero, the function has an inflection point and the concavity changes.

These are just a few examples of the tools we can use to analyze a function. Understanding these concepts can be useful for identifying important features of the function, such as local extrema and points of inflection, which can be useful for solving optimization problems.

In this chapter, we've learned about some of the tools we can use to analyze a function, including finding the critical points, determining the intervals of increase and decrease, and understanding the concavity of the function. These concepts can be useful for identifying important features of a function, such as local extrema and points of inflection, which can be useful for solving optimization problems.

In the next chapter, we will be discussing applications of functions in real-world scenarios, and how we can use our understanding of functions to solve problems in different fields such as physics, engineering, and economics.

Thank you for joining me today. I hope you found this chapter on analyzing functions helpful and informative. I can't wait for our next lesson!

Chapter 5: Applications of Functions

Welcome back, students! In the previous chapter, we learned about analyzing functions and how to understand their critical points, intervals of increase and decrease, and concavity. In this chapter, we'll be exploring how functions can be applied to real-world scenarios and how our understanding of functions can be used to solve problems in various fields such as physics, engineering, and economics.

In physics, functions are used to model a wide range of phenomena, such as the motion of objects and the behavior of systems. For example, in kinematics, the study of motion, we use functions to describe the position, velocity, and acceleration of an object over time. In mechanics, the study of forces and energy, we use functions to describe the behavior of systems under various conditions.

In engineering, functions are used to model and design systems. For example, in electrical engineering, we use functions to describe the behavior of circuits and to design control systems. In civil engineering, we use functions to model the behavior of structures, such as bridges and buildings, under various loads and conditions.

In economics, functions are used to model and understand how different factors affect the economy. For example, supply and demand functions describe how the quantity of a good that firms are willing to produce and the quantity that consumers are willing to purchase are related to the price of the good.

Functions also have applications in computer science, biology, and many other fields. In many cases, the mathematical models used in these fields are based on functions and their properties.

In this chapter, we've learned about the wide range of applications that functions have in the real world, in fields such as physics, engineering, economics, and many others. We've seen that our understanding of functions and their properties can be used to model and understand complex phenomena and to solve problems in these fields.

In conclusion, functions are a fundamental concept in mathematics and it’s really important to understand the basic concepts and how to apply them in real-world scenarios. I hope this book has helped you to understand the key concepts of functions and its applications in a clear and friendly way.

Thank you for joining me throughout this journey of exploring functions. I hope you found it informative and helpful.

Chapter 6: Conclusion and Further Study

In this book, we've covered the basic concepts of functions and explored various types of functions, properties, transformations, and methods for analyzing them. We've also seen the wide range of applications that functions have in the real world and how our understanding of them can be used to model and understand complex phenomena, and to solve problems in various fields such as physics, engineering, and economics.

We have learned that functions are a fundamental concept in mathematics and it is important to understand the basic concepts and how to apply them in real-world scenarios. I hope this book has helped you to understand the key concepts of functions and its applications in a clear and friendly way.

As you continue to study mathematics, you'll find that functions are a fundamental tool for solving many types of problems, and your understanding of them will become more sophisticated and nuanced. I recommend that you continue to practice working with functions and to seek out additional resources to help deepen your understanding.

There are many resources available to help you further your study of functions, such as textbooks, online tutorials, and interactive simulations. You can also explore the applications of functions in various fields, such as physics, engineering, economics, and computer science.

In conclusion, I want to thank you for reading this book, and I hope it has been helpful in your understanding of functions. Remember to always practice, keep an open mind and don't hesitate to ask for help when needed.

Good luck with your studies!