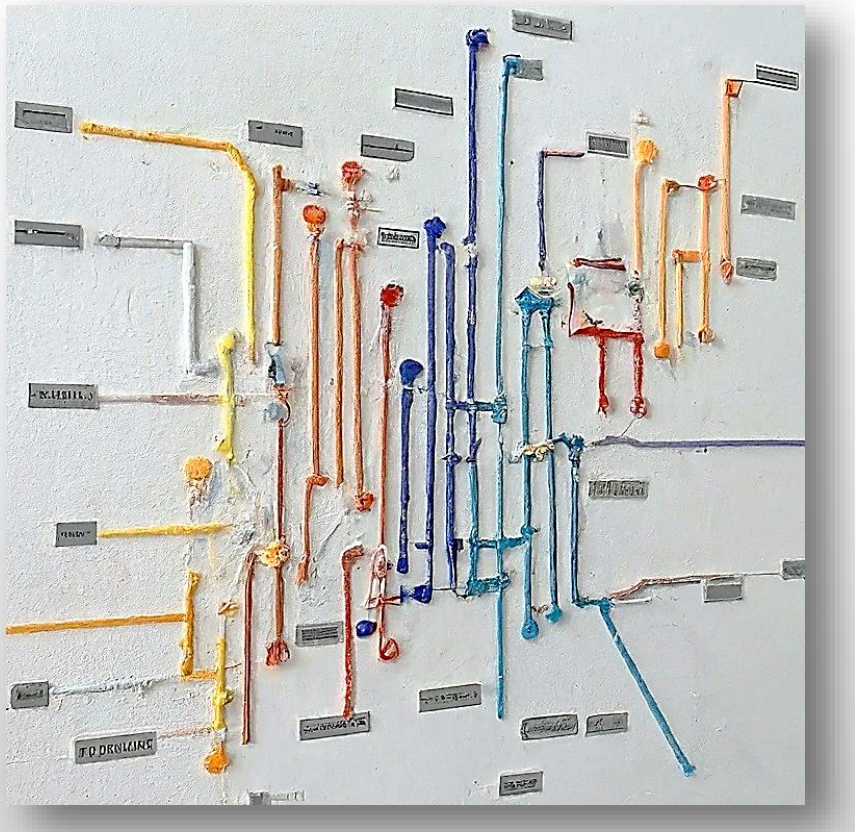
Bölüm 4: Çizge Algoritmaları

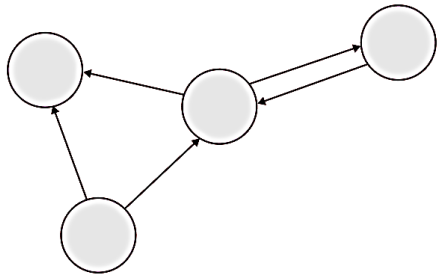
Algoritmalar

Çizge Algoritmaları

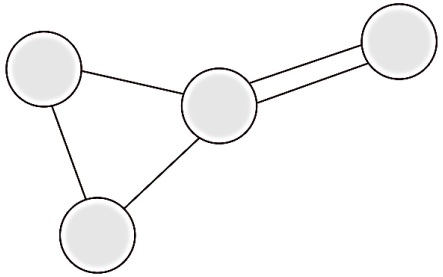
- Dünya aslında bir ağ gibidir.
  - Şehirler yollarla,
  - İnsanlar ilişkilerle,
  - Bilgisayarlar kablolarla birbirine bağlıdır.
- Çizge algoritmaları bu ağları inceler ve anlamlandırır.



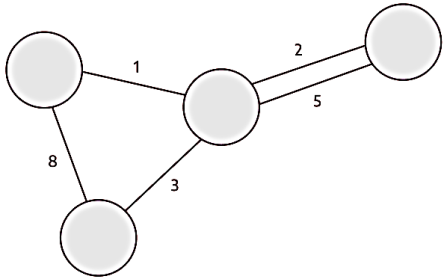
Çizge Türleri



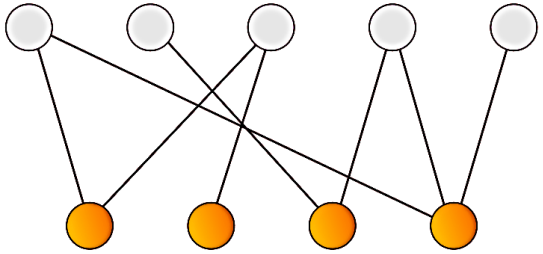
Directed graph



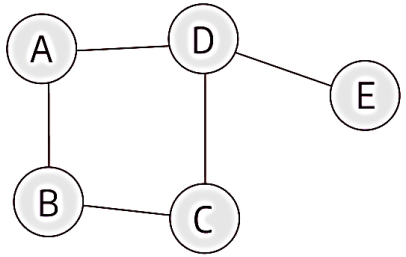
Undirected



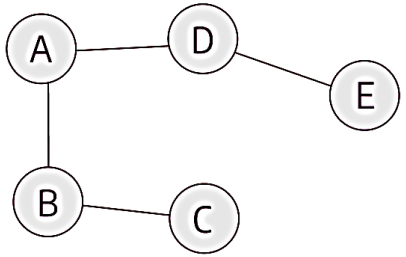
Weighted



Bipartite graph



Cyclic graph



Acyclic graph

Çizge Algoritmaları

- Birbirine bağlı noktalar (düğüm) ve bu noktaları birleştiren çizgiler (kenar) ile temsil edilen ağ yapılarını inceler.
- Ağlarda en kısa yolu hesaplama, gruplama gibi işlemleri gerçekleştirir.
- Sosyal ağlar, harita uygulamaları, navigasyon gibi birçok alanda kullanılır.

Çizge Algoritmalarının Çeşitleri

- Farklı çizge algoritmaları, farklı işlemler için kullanılır.
- Derinlik Öncelikli Arama (DFS):
  - Bir düğümden başlar, dallanarak tüm ağı gezer.
- Genişlik Öncelikli Arama (BFS):
  - Bir düğümden başlar, katman katman tüm ağı gezer.
- Dijkstra Algoritması:
  - Başlangıç düğümünden diğer düğümlere en kısa yolları bulur.
- Kruskal Algoritması:
  - Bir ağı minimum maliyetle birbirine bağlayan kenarları seçer.

Çizge Algoritmaları

- DFS bir labirentten çıkış yolu ararken kullanılabilir.
- BFS bir haberin tüm şehire yayılma sürecini modelleyebilir.
- Dijkstra en kısa sürede teslimat yapmak için kullanılabilir.

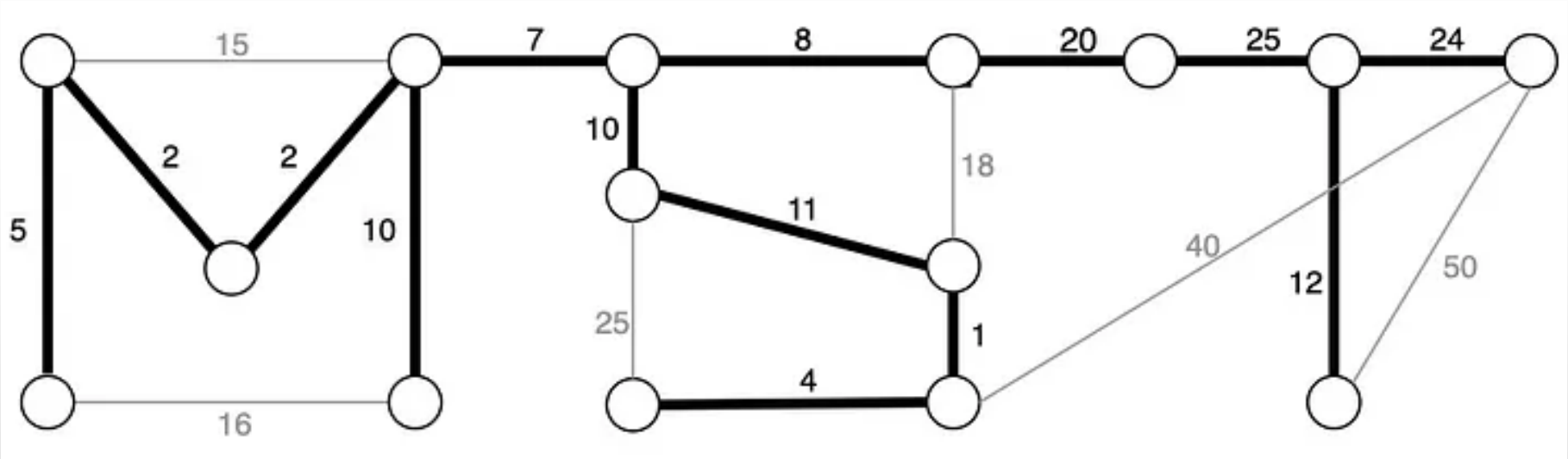
Çizge Algoritmaları

- Çizge gezinme algoritmaları (*Graph traversal*)
- En kısa yol algoritmaları (*Shortest path*)
- Minimum kapsayan ağaç algoritmaları (*Minimum spanning tree*)
- Ağ akış algoritmaları (*Network flow*)

Minimum Kapsayan Ağaç Algoritmaları

- Çizgedeki,
  - tüm düğümleri birbirine bağlayan ve
  - toplam kenar ağırlığının en az olduğu alt ağaçtır.
- *Kruskal*, kenarları ağırlıklarına göre sıralar ve döngü oluşturmayan kenarları seçerek ağacı oluşturur.
- *Prim*, başlangıç düğümünden başlayarak, her adımda en düşük ağırlıklı kenarı seçerek ağacı büyütür.

Minimum Kapsayan Ağaç (MST)



Kruskal Algoritması

- Çizgede tüm düğümleri birbirine bağlayan en kısa ağırlıklı ağacı oluşturur.
- Açgözlü (greedy) bir yaklaşım kullanır.
- Joseph Kruskal tarafından geliştirilmiştir.

Algoritma İlkeleri

- Ağırlıklı çizge üzerinde çalışır.
- Tüm düğümleri en küçük ağırlıklı kenarları kullanarak birleştirir.
- Döngü oluşturmadan, minimum kapsayan ağacı oluşturur.
- Başlangıçta, her düğüm ayrı bir ağacı temsil eder.
  - Adım adım bu ağaçlar birleştirilir.

Algoritma Adımları

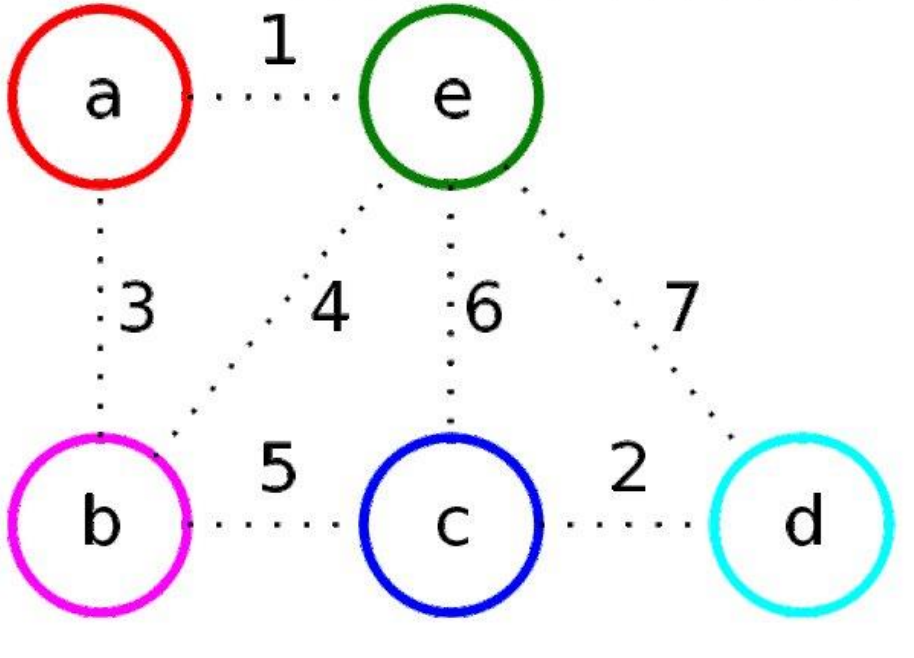
- Adım 1: Çizge içindeki tüm kenarlar ağırlıklarına göre sıralanır.
- Adım 2: Sıralı kenarlar arasından en küçük ağırlıklı kenar seçilir.
- Adım 3: Seçilen kenar, döngü oluşturmuyorsa, ağaç içine eklenir.
- Adım 4: Eğer seçilen kenar, farklı ağaçlara ait düğümleri birleştiriyorsa, bu kenar ağaç içine eklenir ve ağaçlar birleştirilir.

Karmaşıklık Analizi

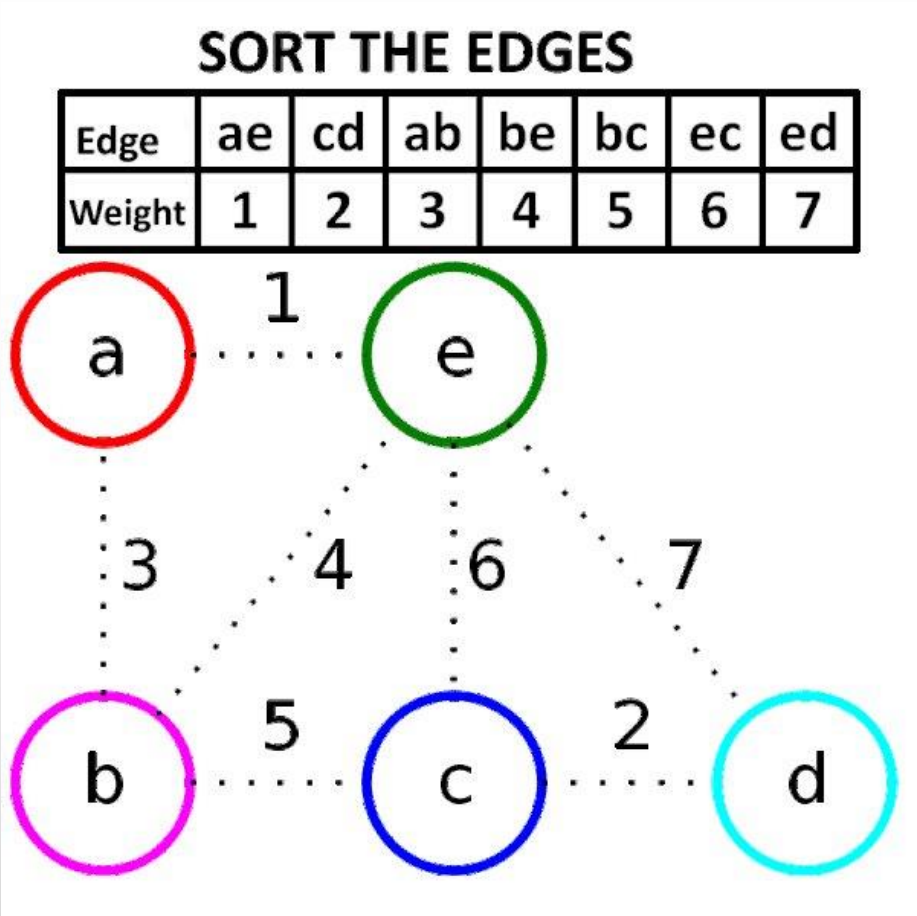
- Kenarların ağırlıklarına göre sıralanması:
  - *O(E log E)*
- Birleştirme-bulma (union-find) işlemleri:
  - *O(log E)* (amortize edilmiş)

Kruskal

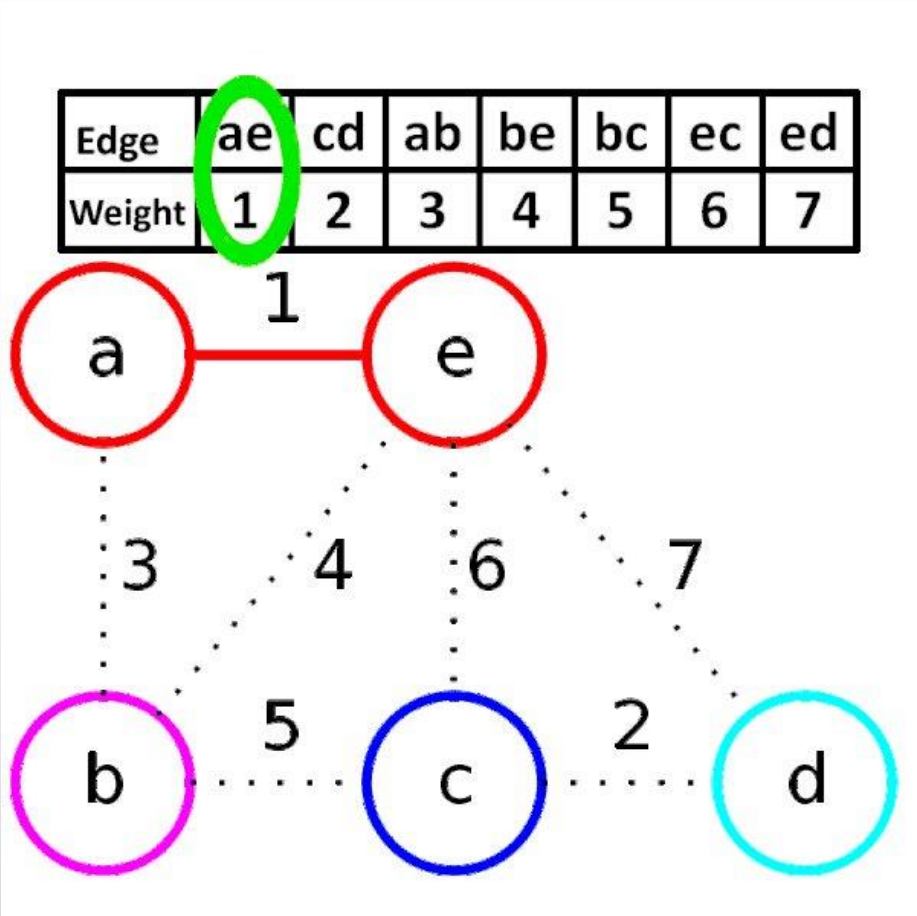
Edge	ab	ae	bc	be	cd	ed	ec
Weight	3	1	5	4	2	7	6



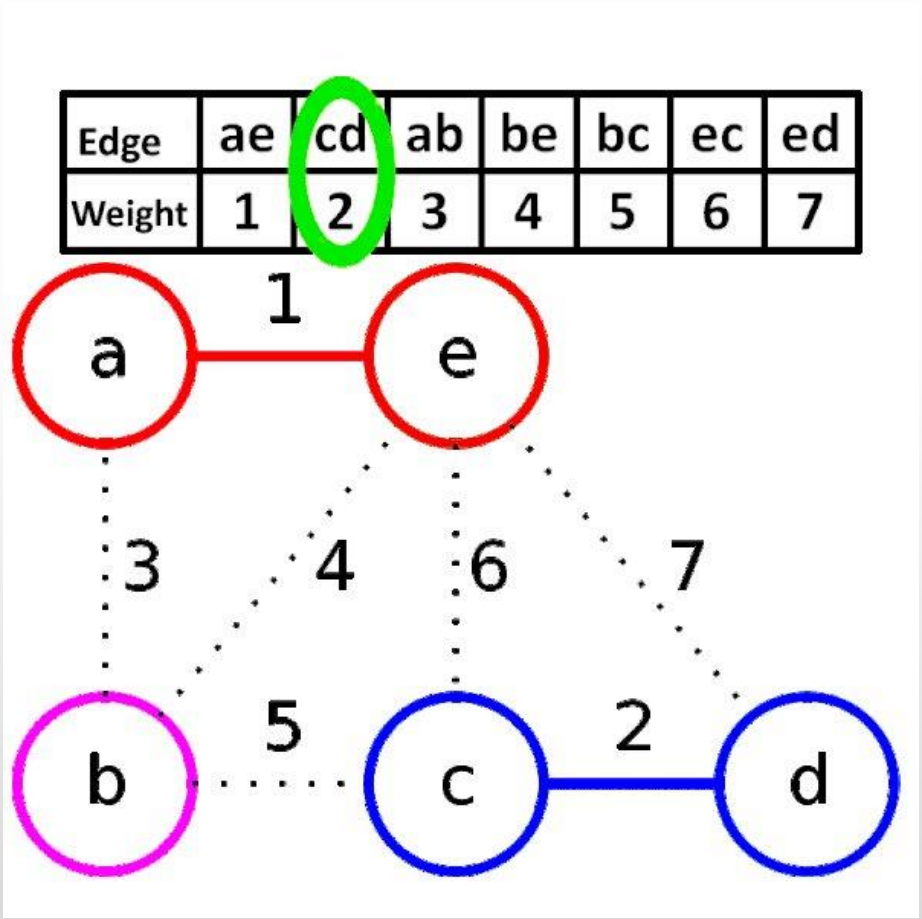
Kruskal



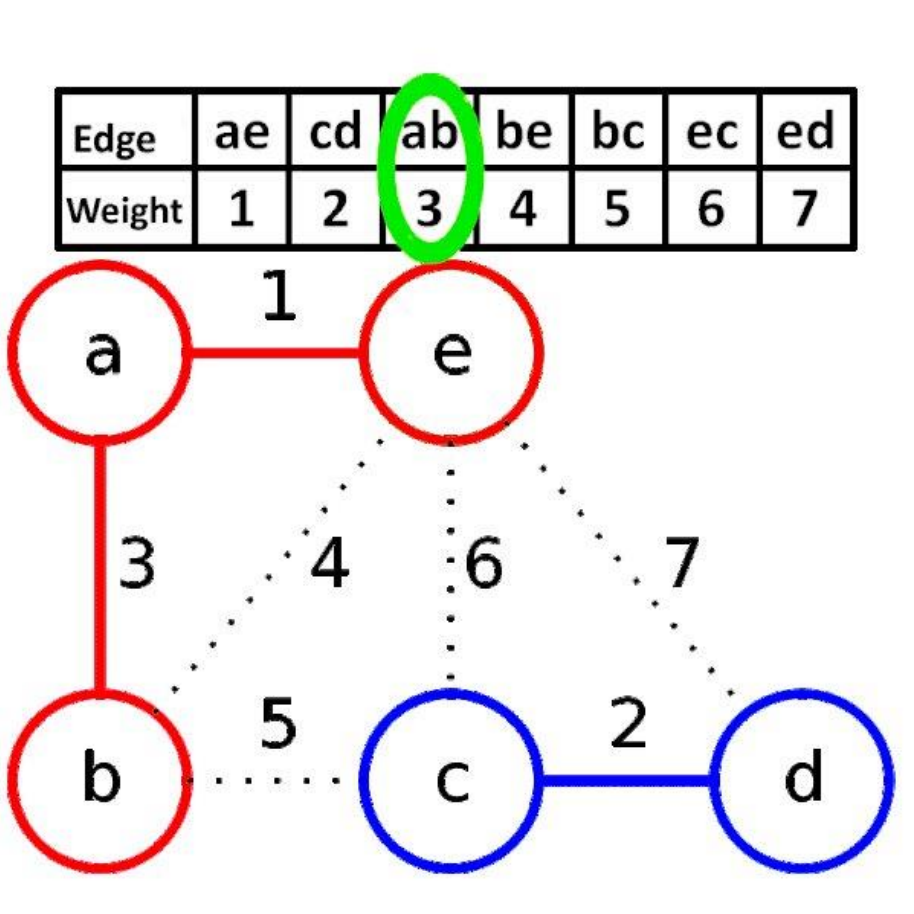
Kruskal



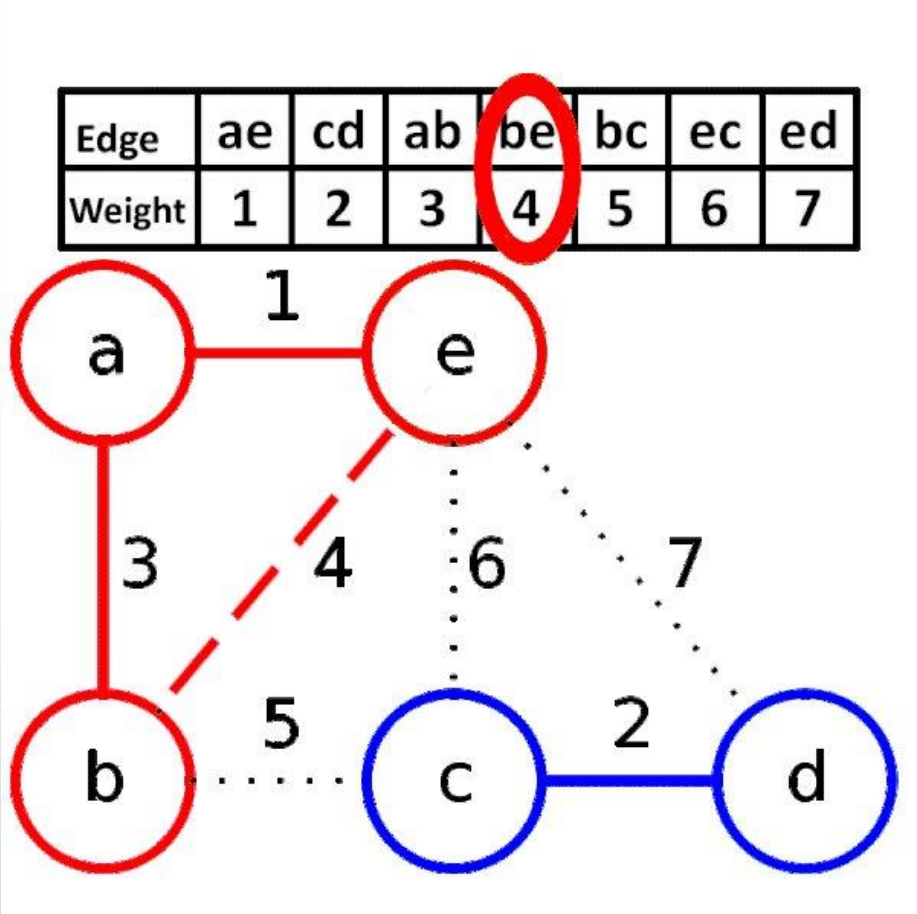
Kruskal



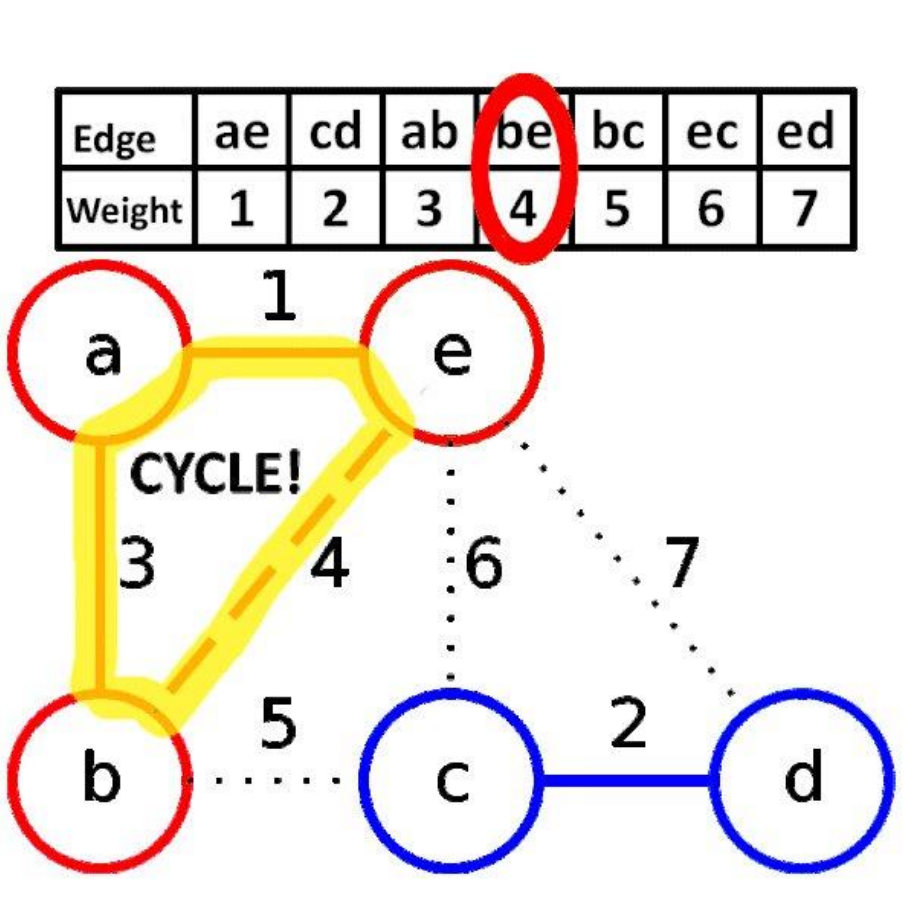
Kruskal



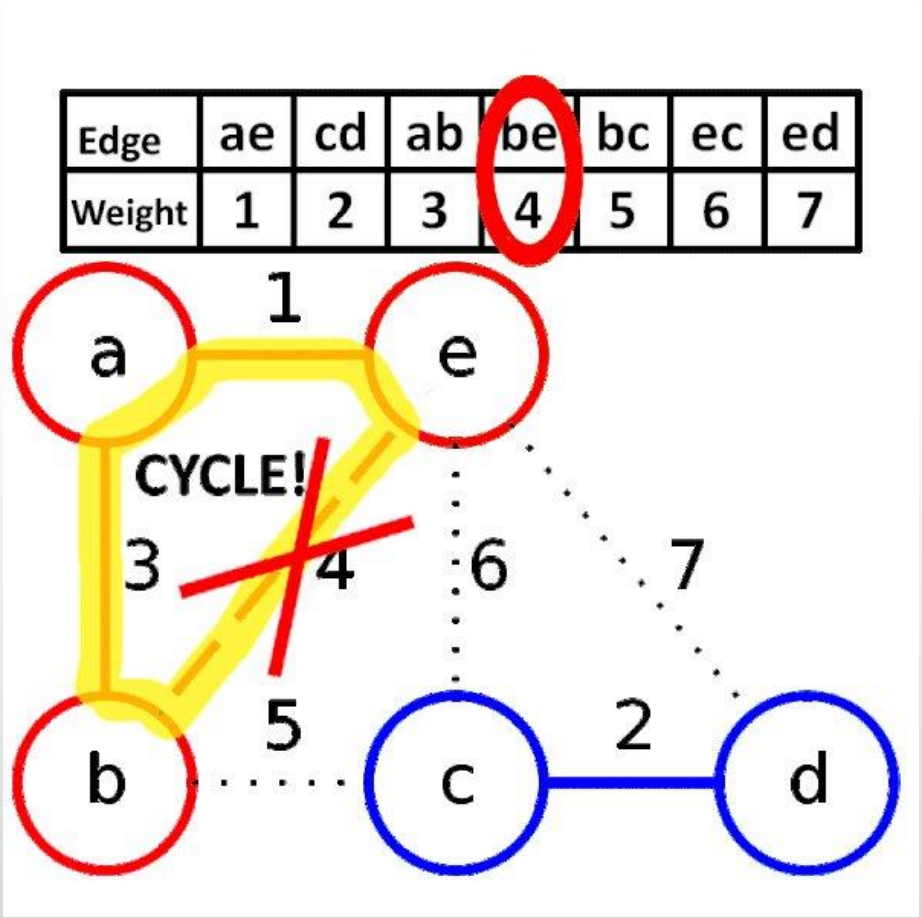
Kruskal



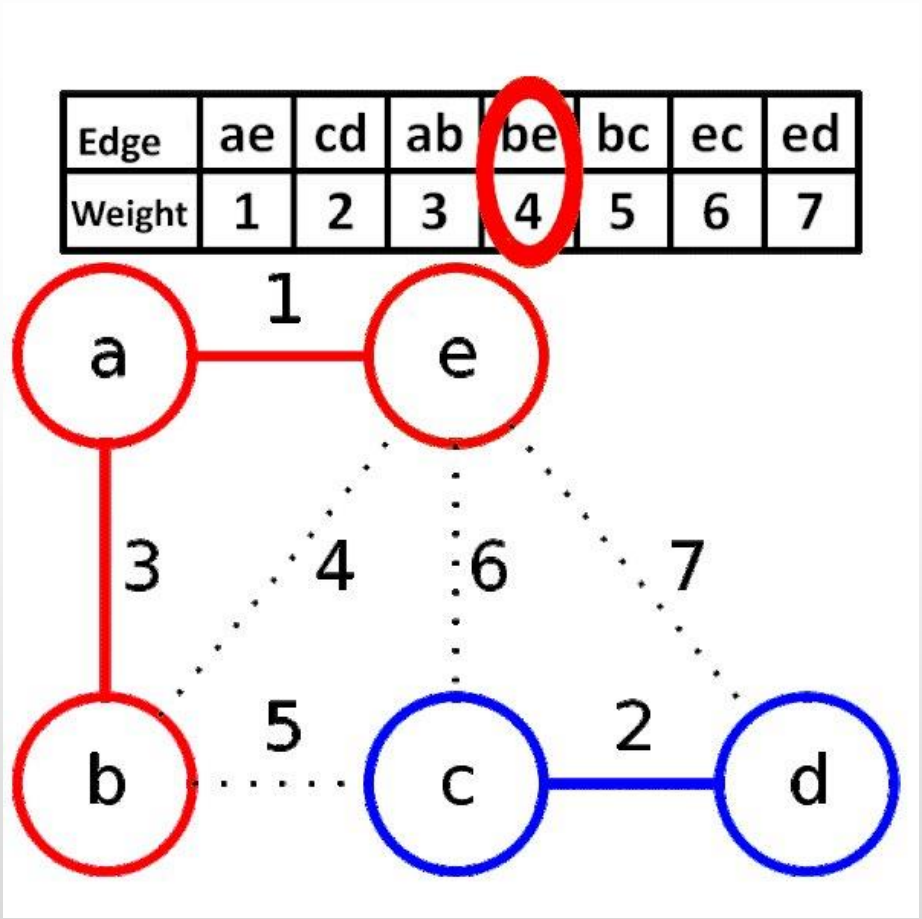
Kruskal



Kruskal



Kruskal



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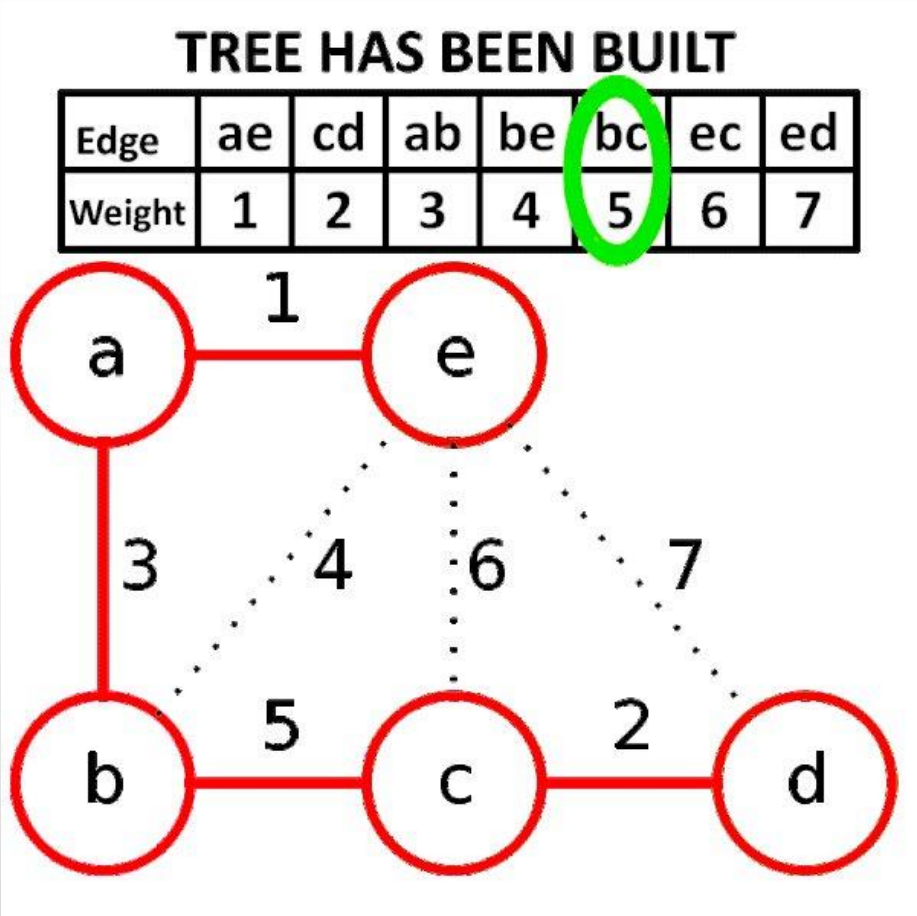
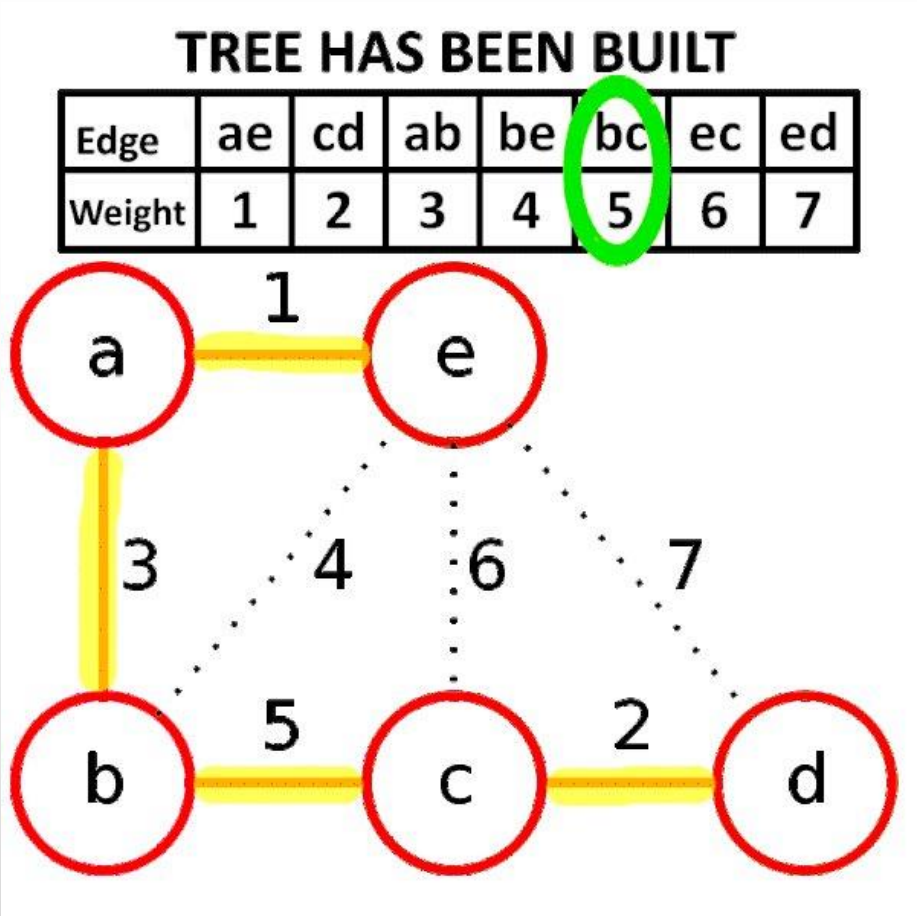
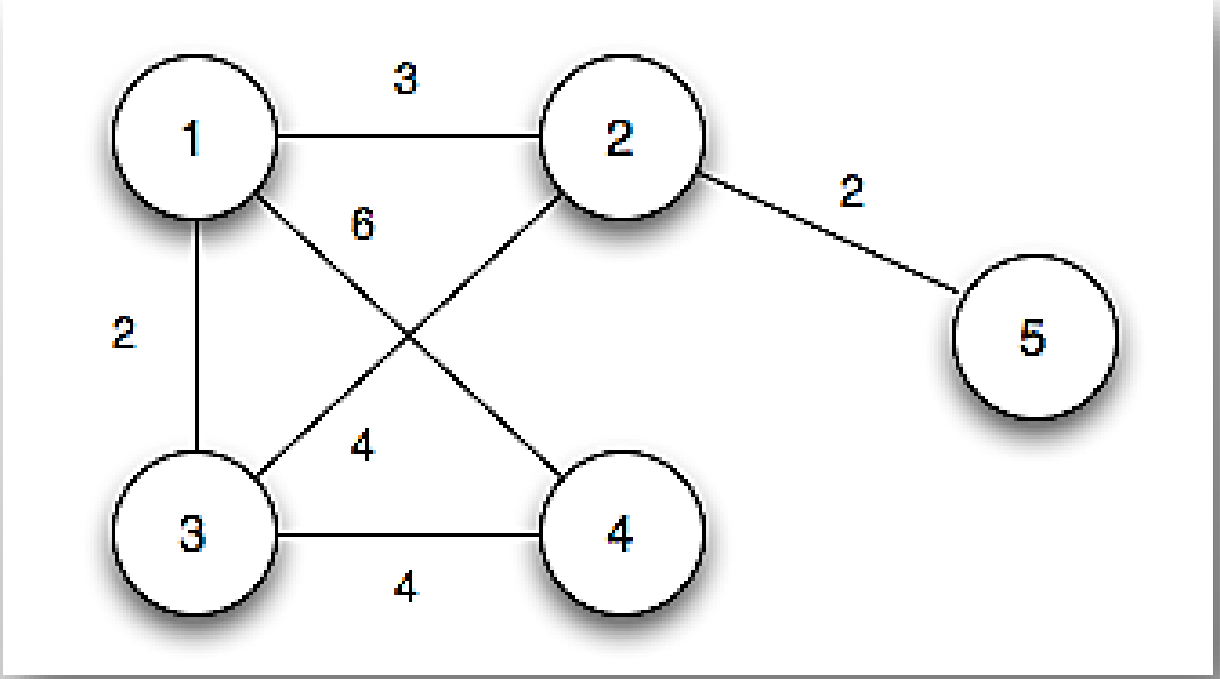


Kruskal



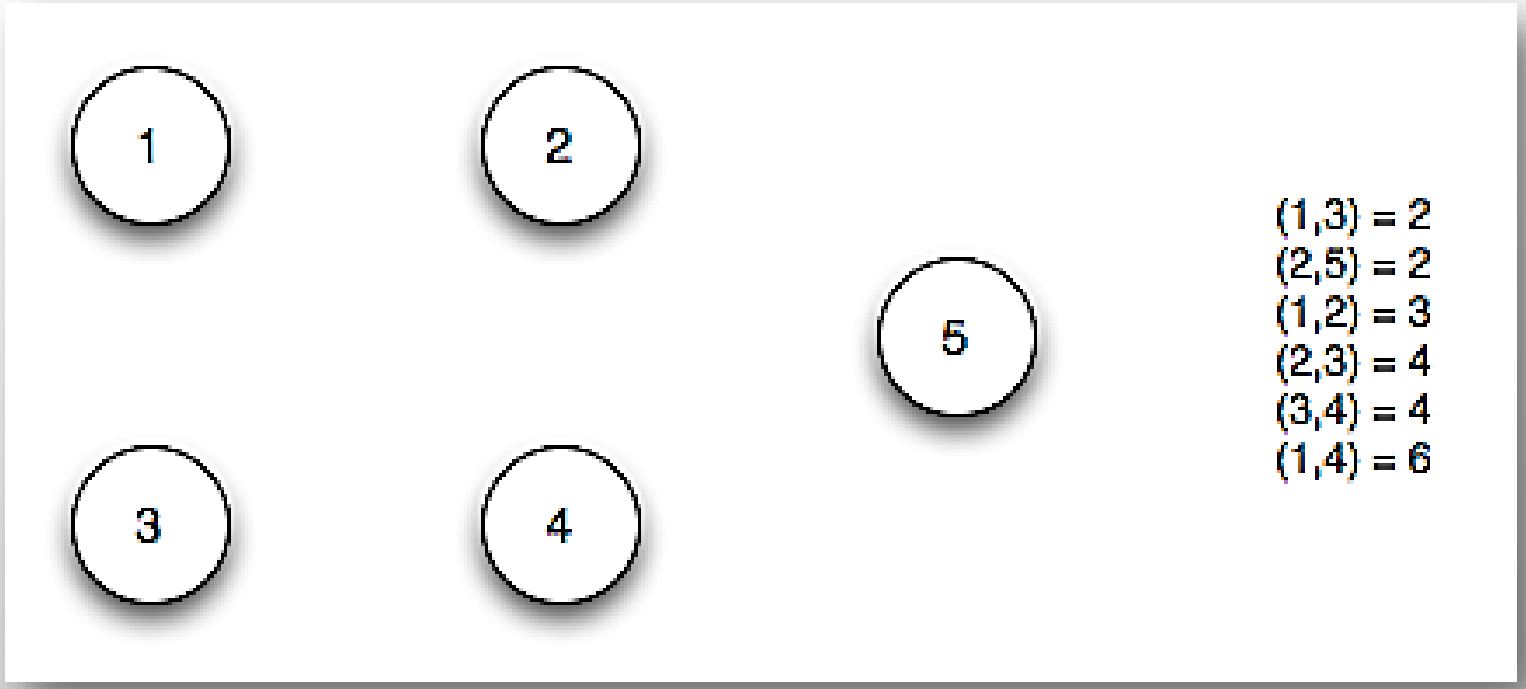
Örnek

- Aşağıdaki yönsüz çizge verilsin.



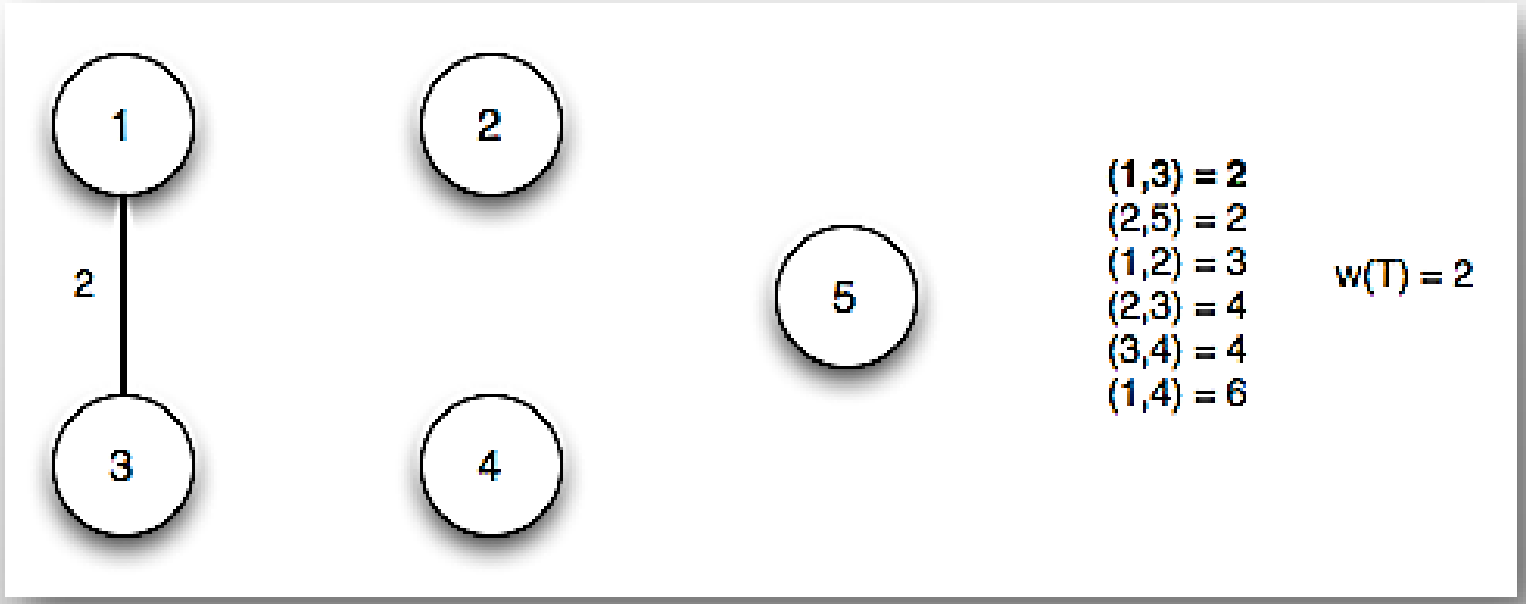
İlklendirme Aşaması

- Her bir düğüm ayrı bir ağaç yapılır.
- Kenarlar ağırlıklarına göre sıralanır.



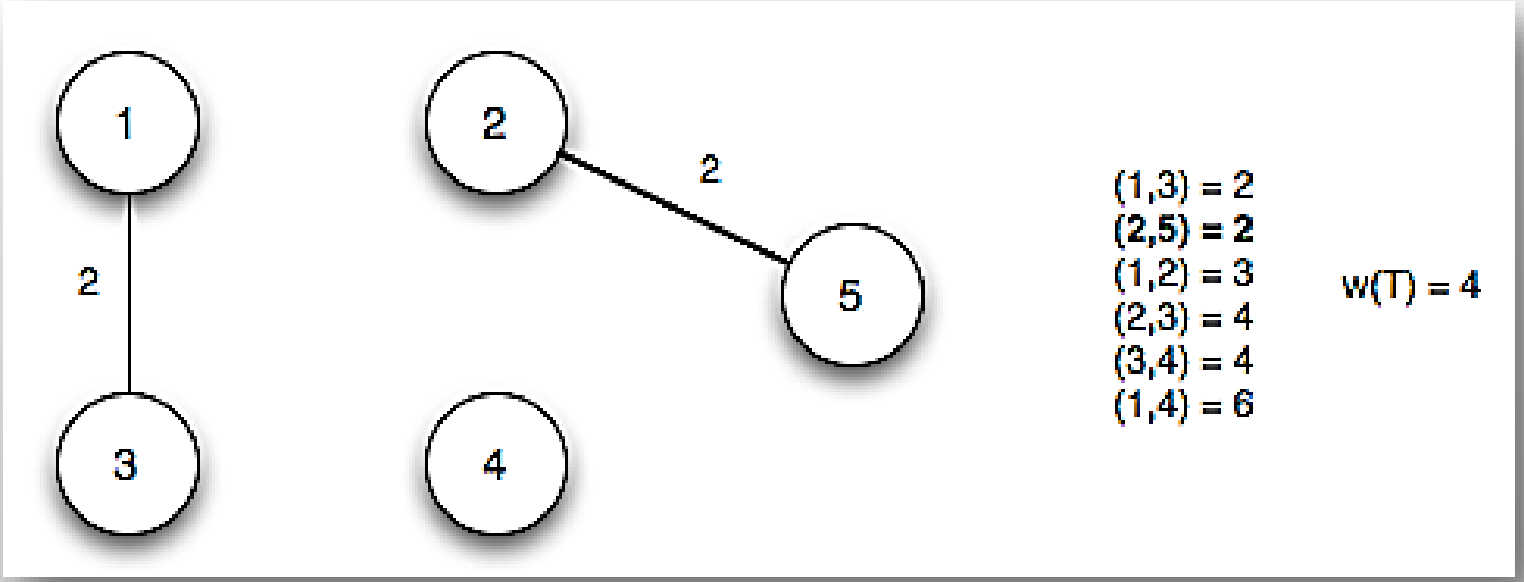
Adım 1

- (u1,u3) kenarını ekle
- v1 ve v3 birleştir



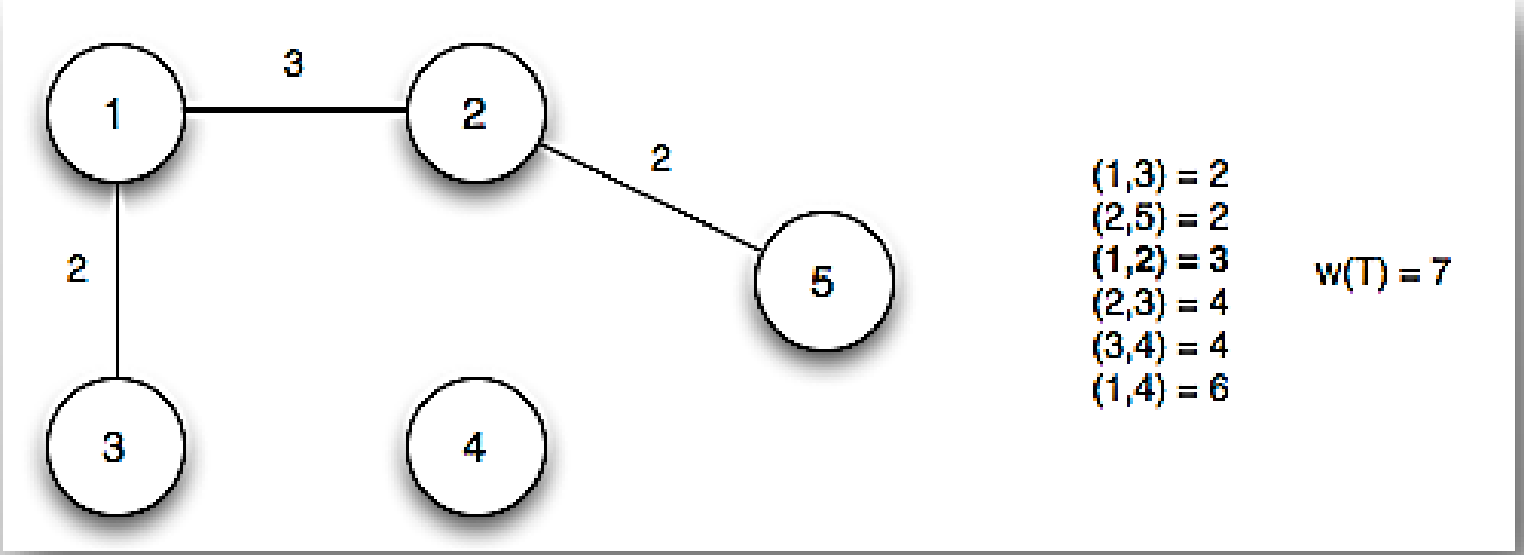
Adım 2

- (u2,u5) kenarını ekle.
- v2 ve v5 birleştir.



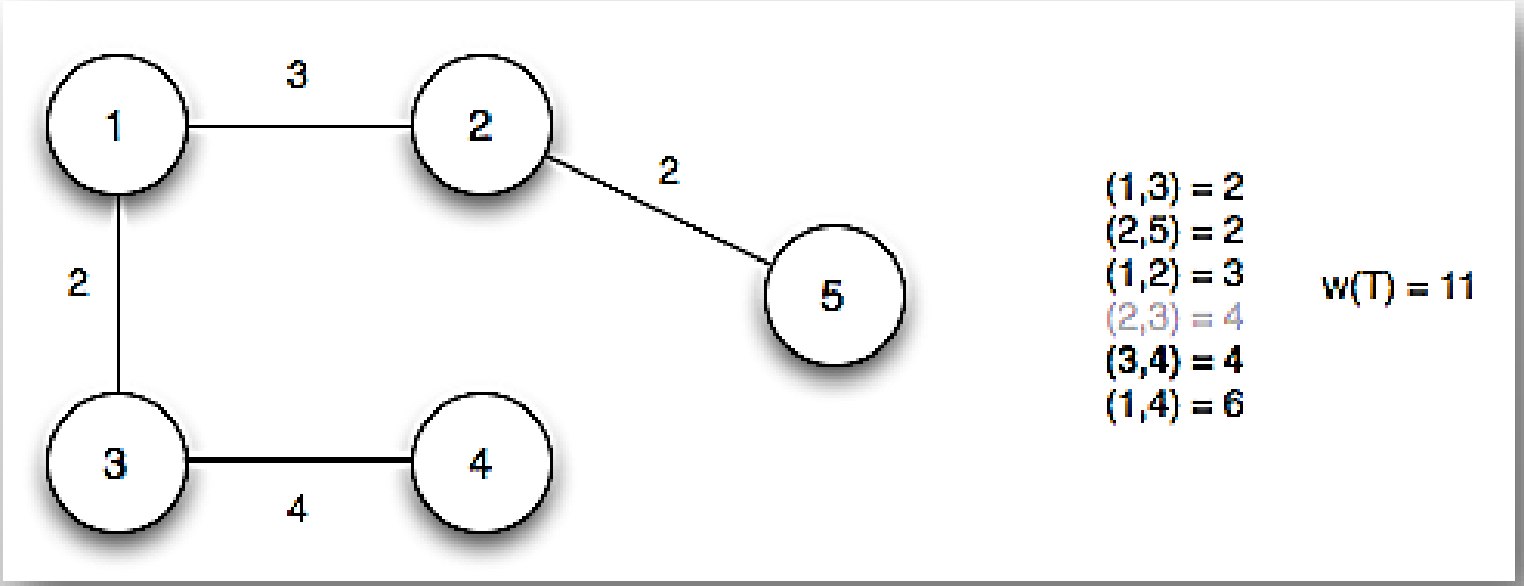
Adım 3

- (u1,u2) kenarını ekle.
- Adım 1 ve Adım 2'de oluşan iki ağacı birleştir.



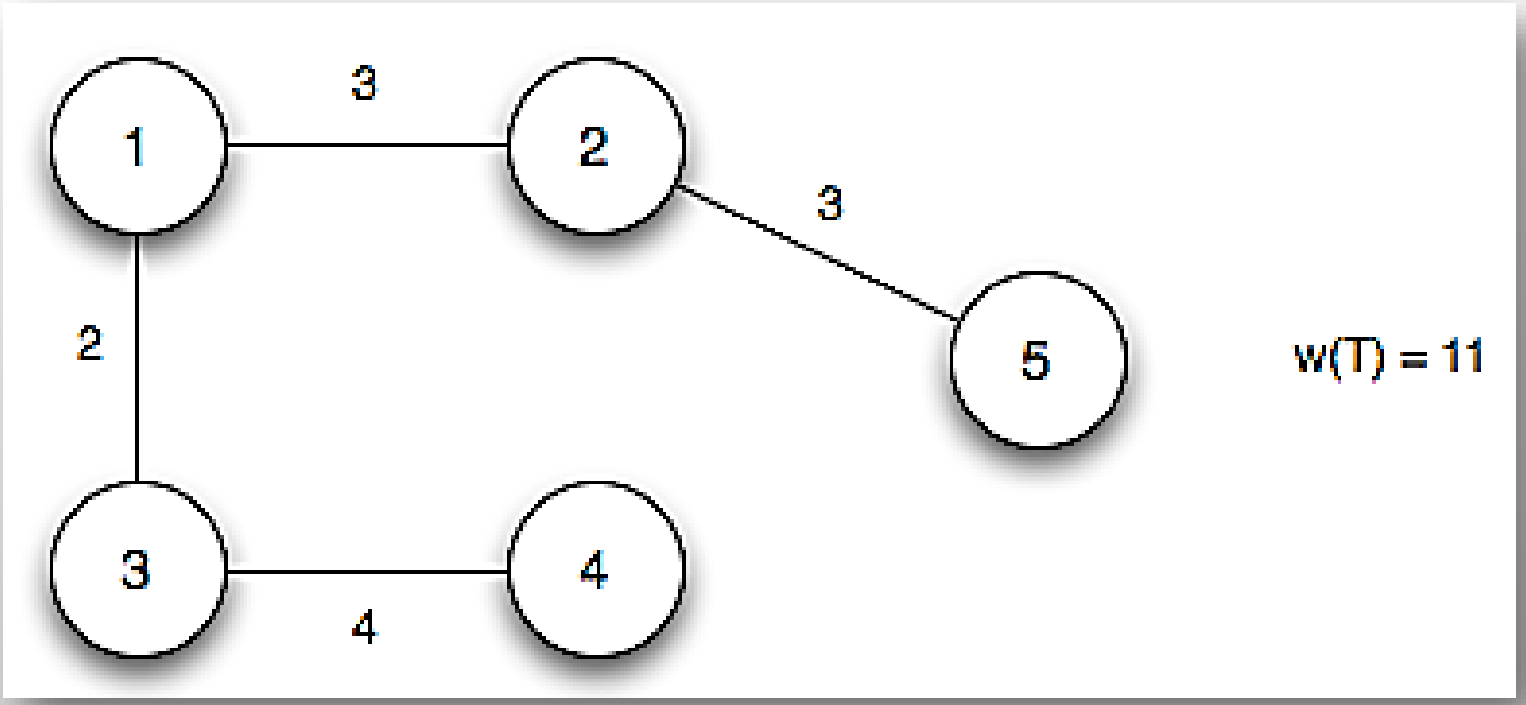
Adım 4

- (u3,u4) kenarını ekle, v4 birleştir.
- (u2,u3) ayrı ağaçları birleştirmiyor!



Son Durum

- (u1,u4) kenarı ayrı ağaçları birleştirmiyor!



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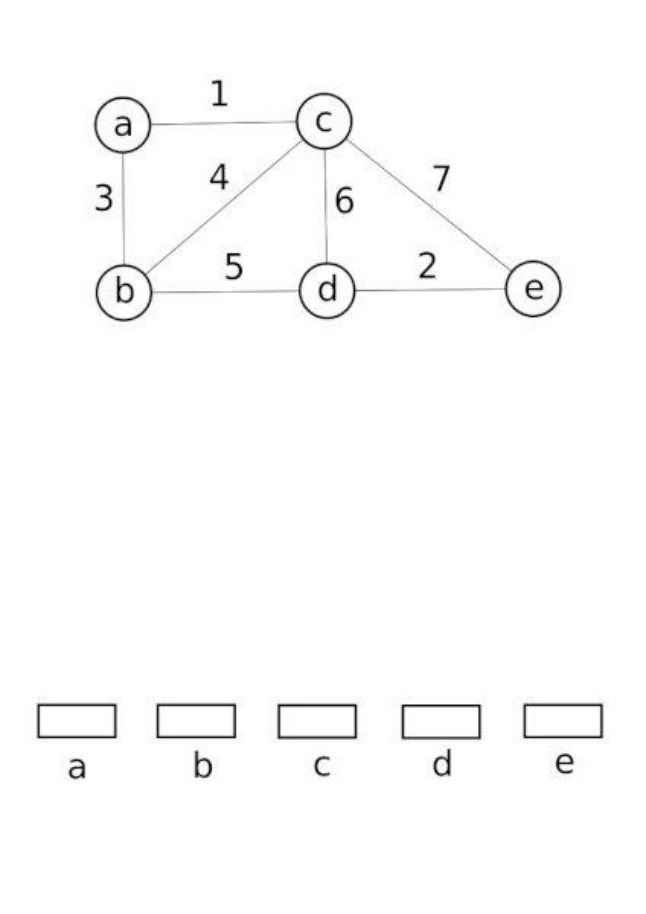

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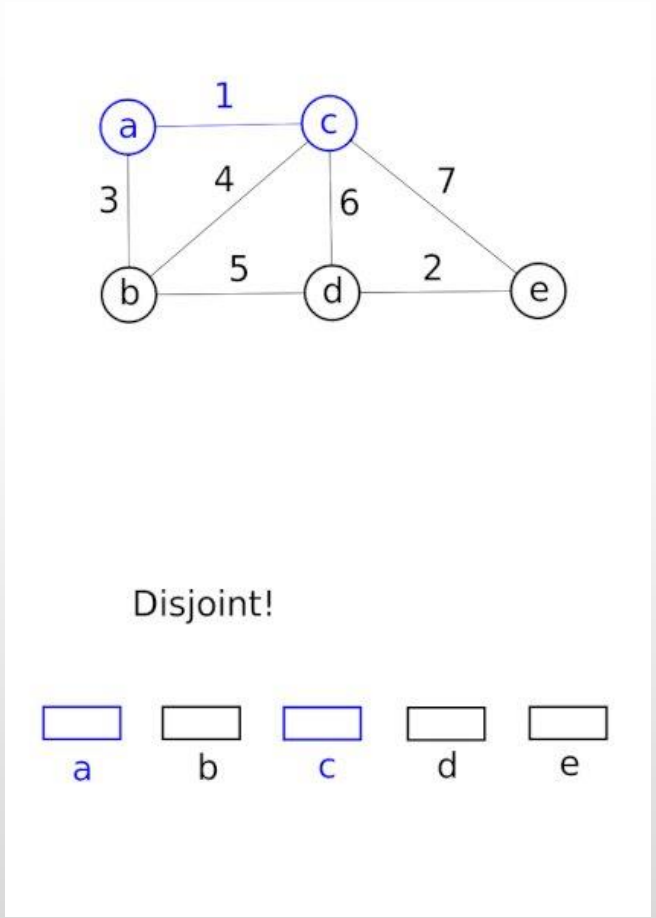

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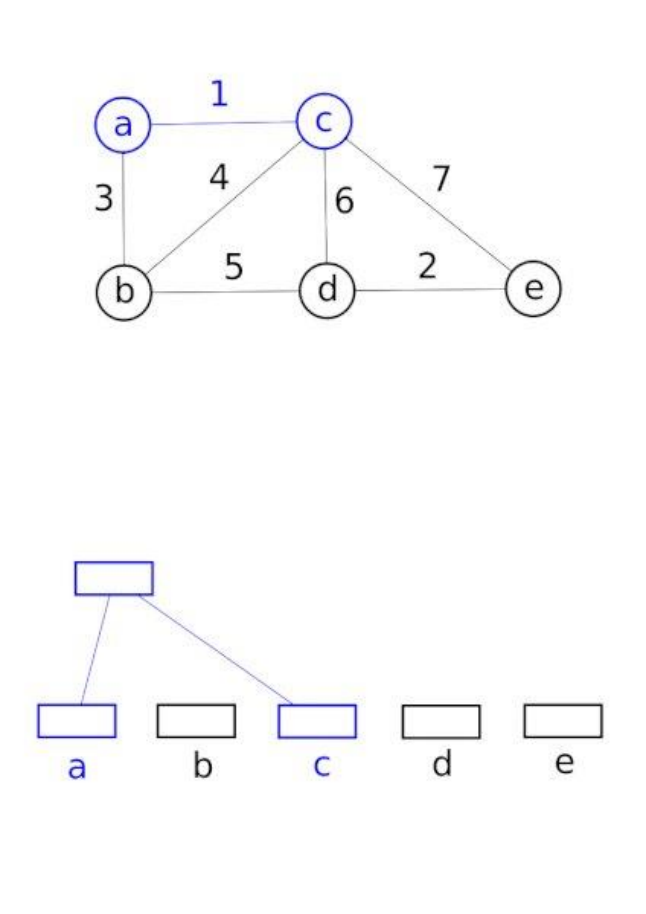

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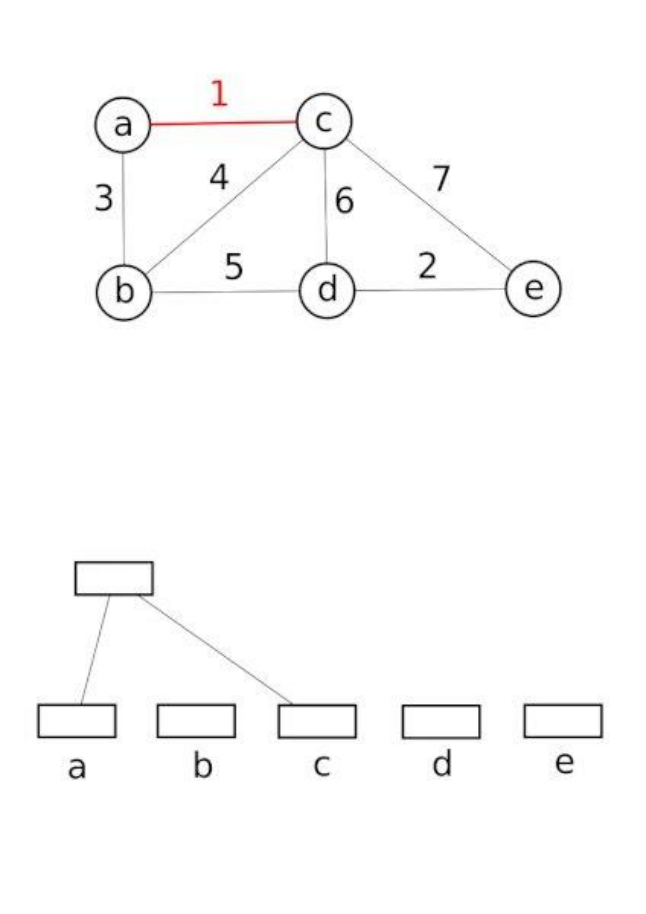

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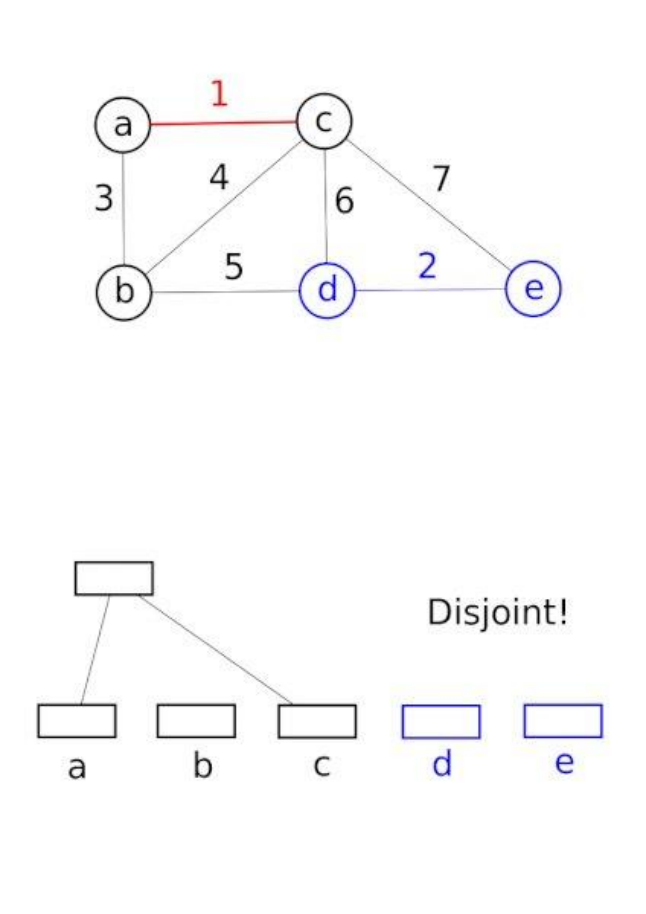

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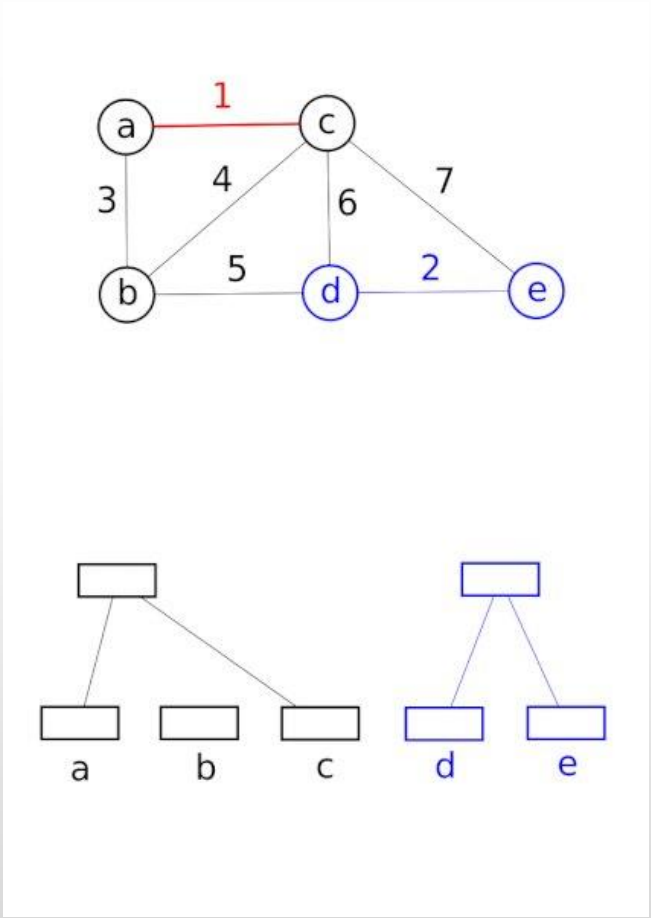

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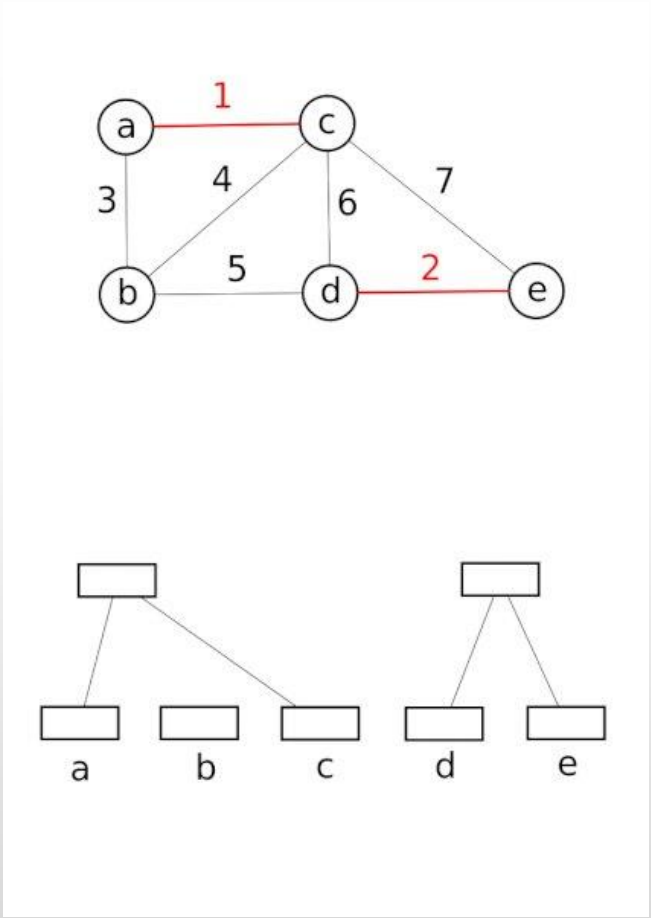

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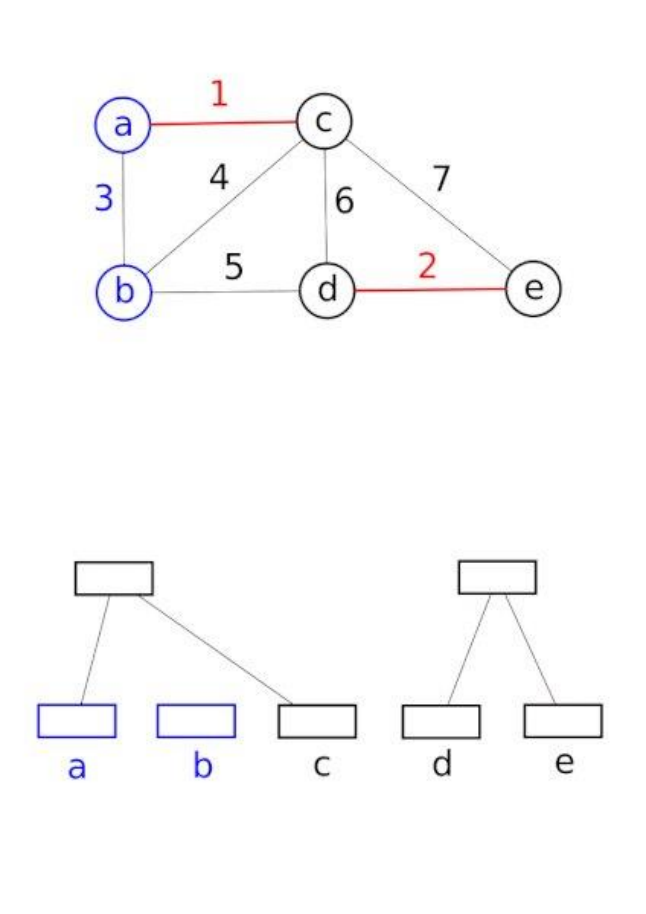

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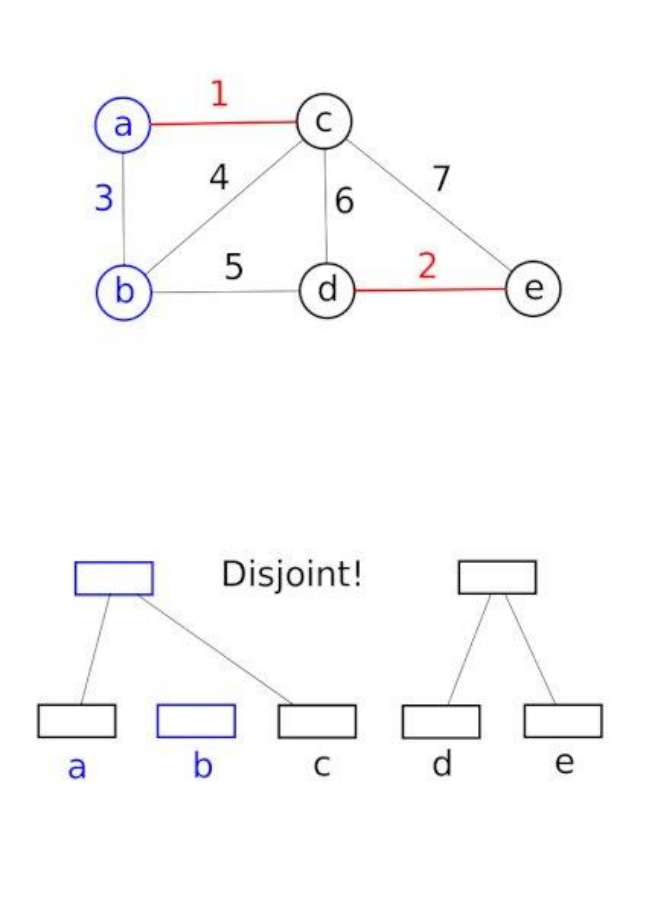

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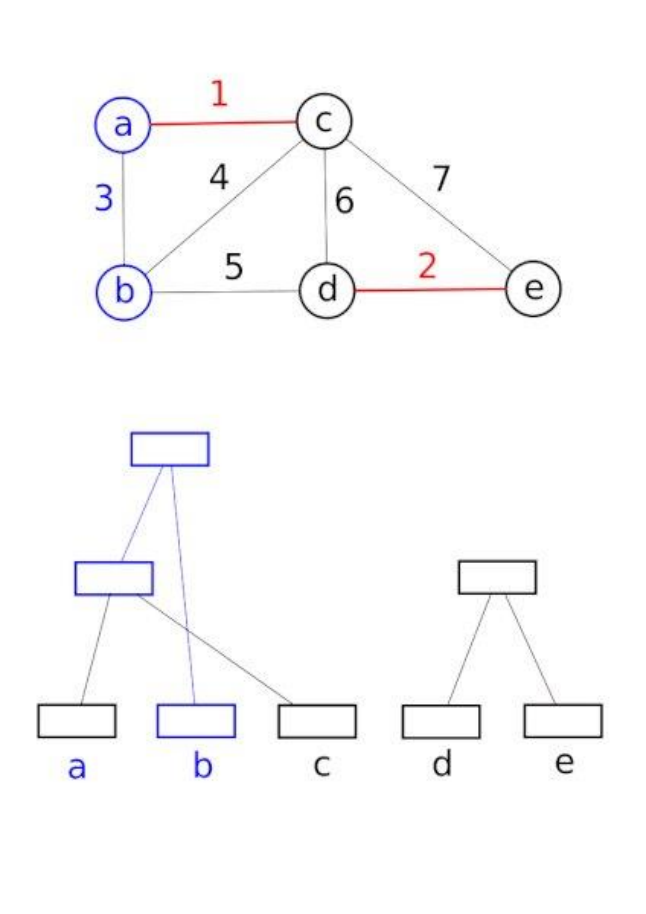

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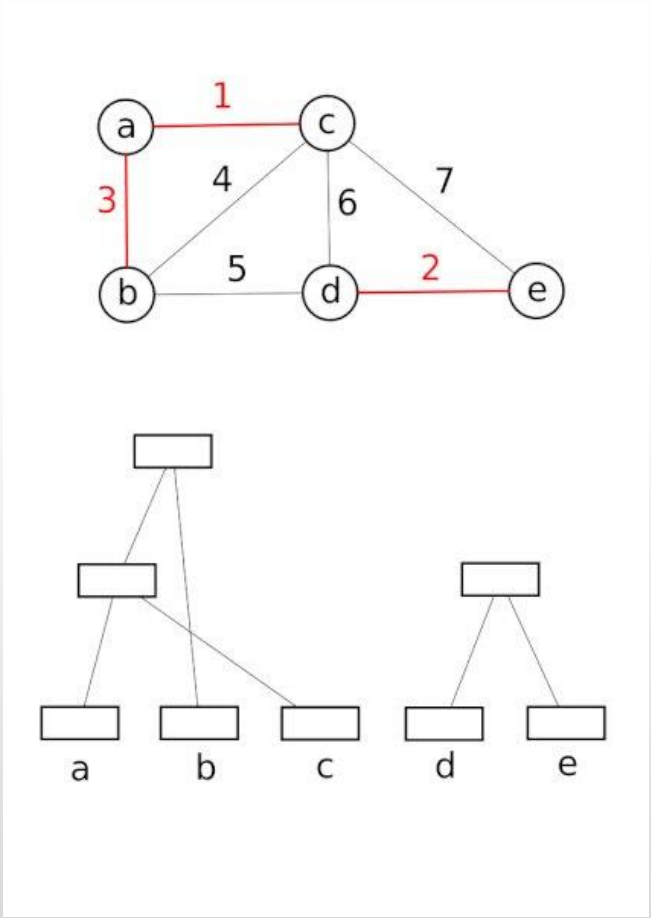

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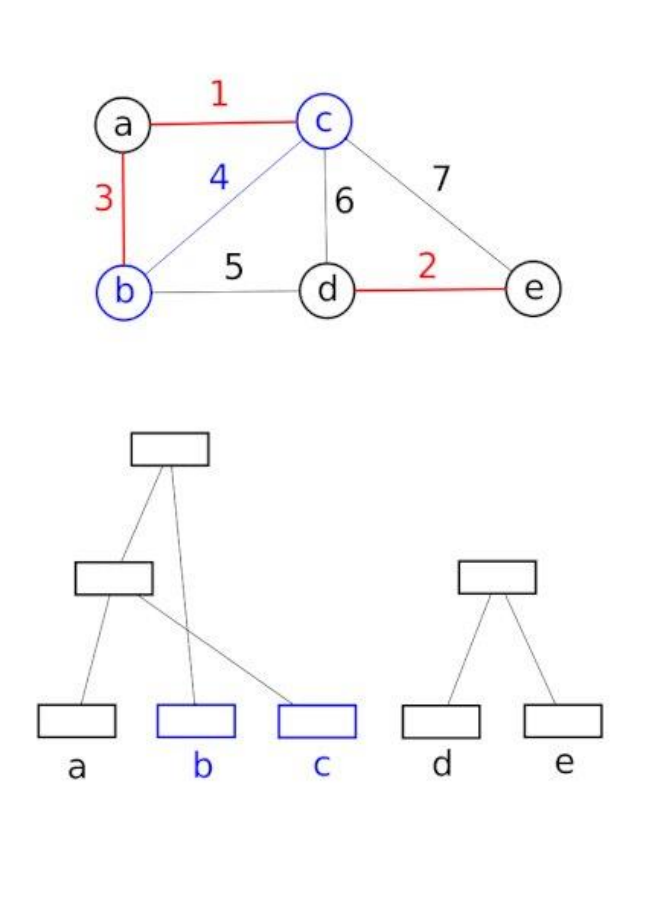

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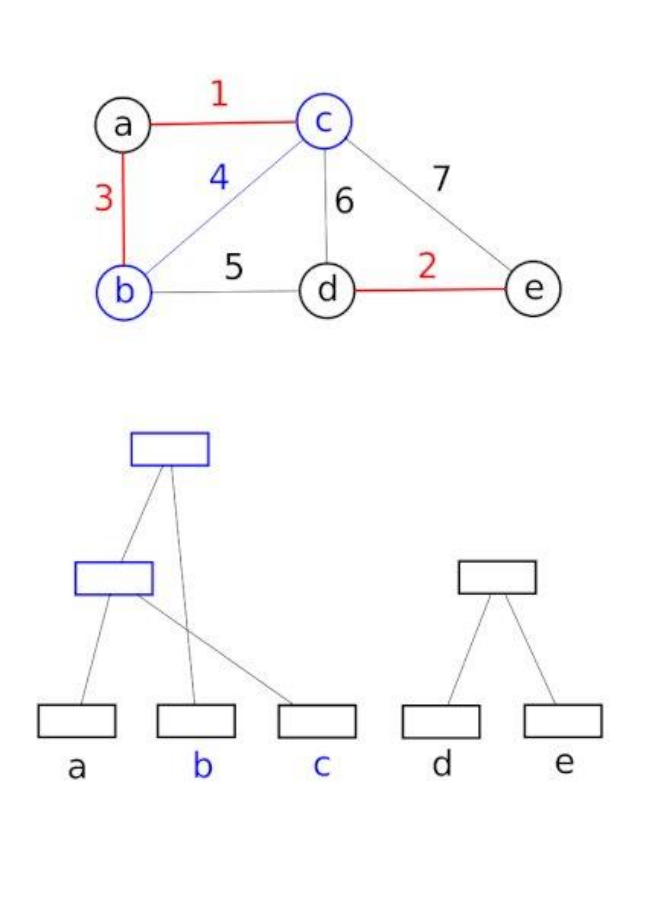

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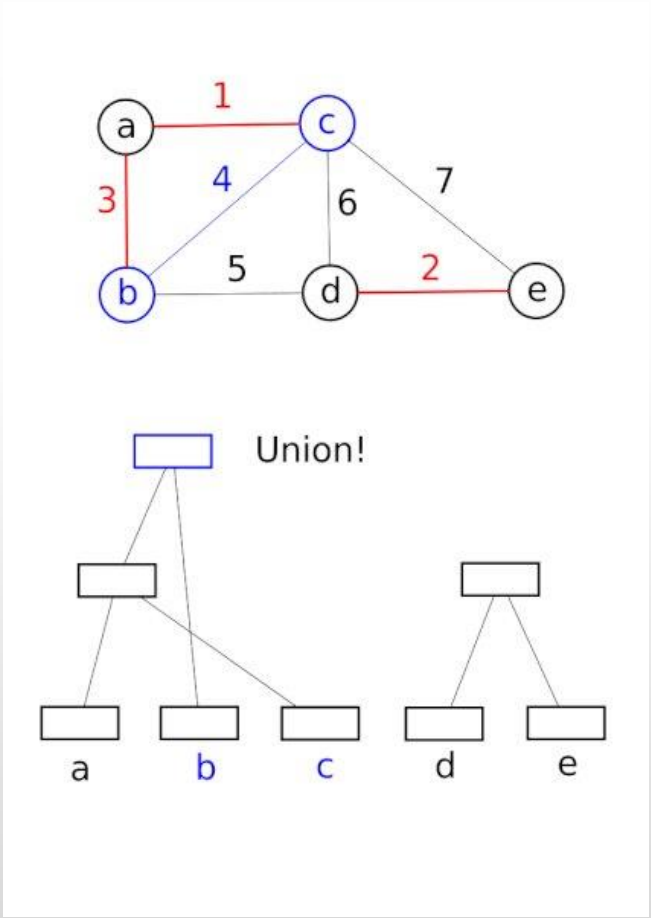

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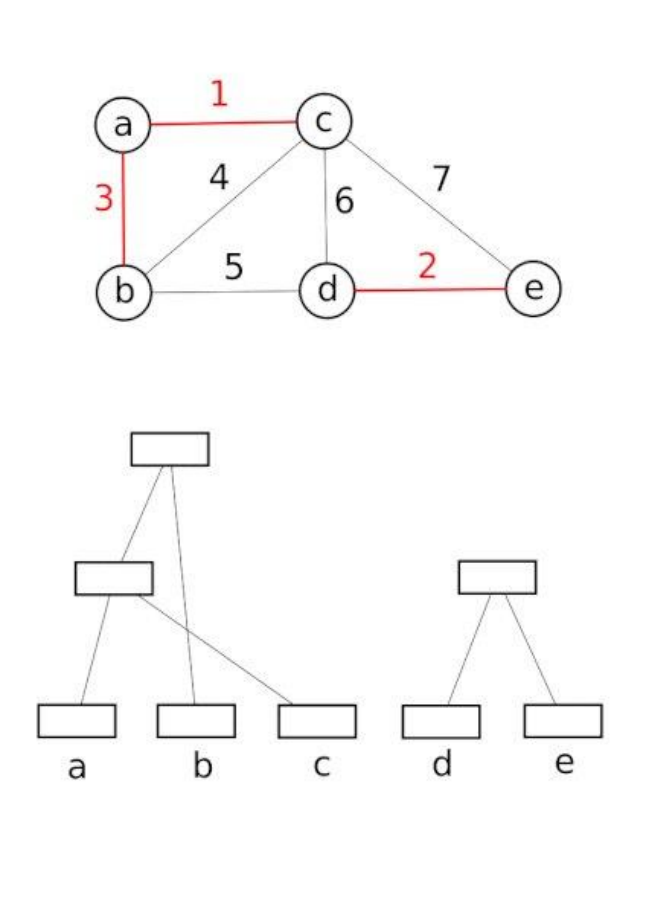

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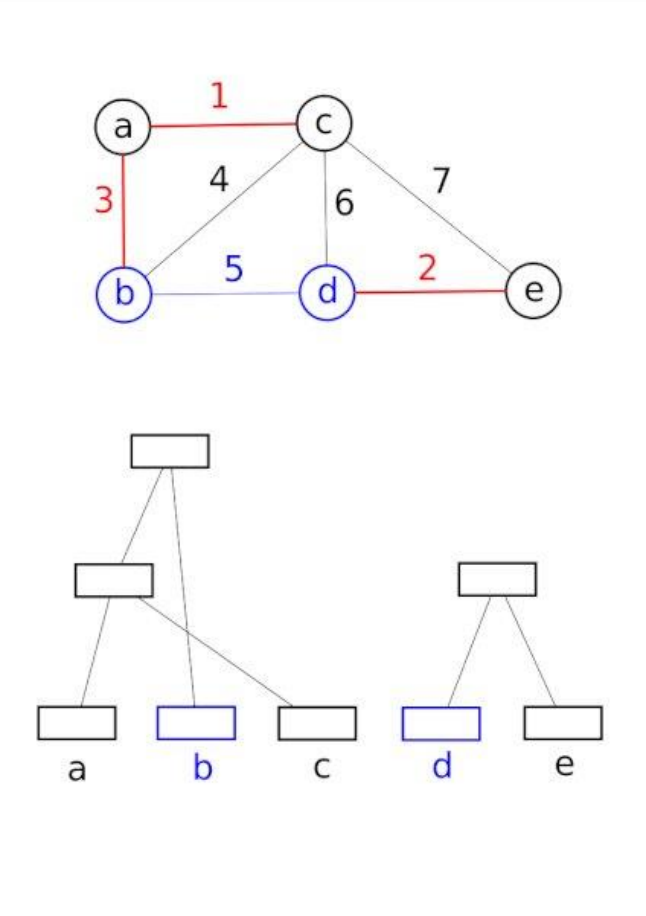

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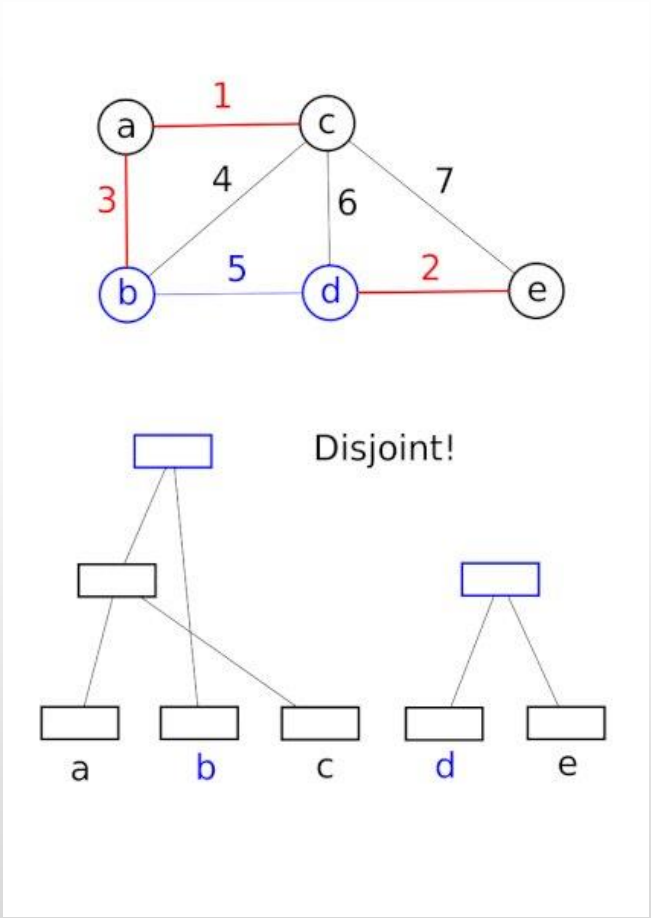

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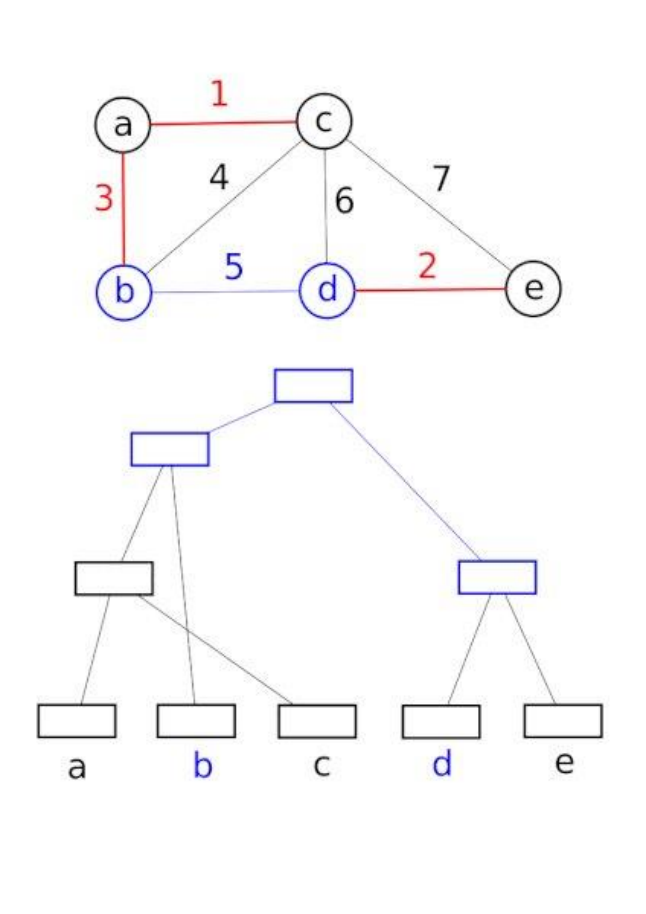

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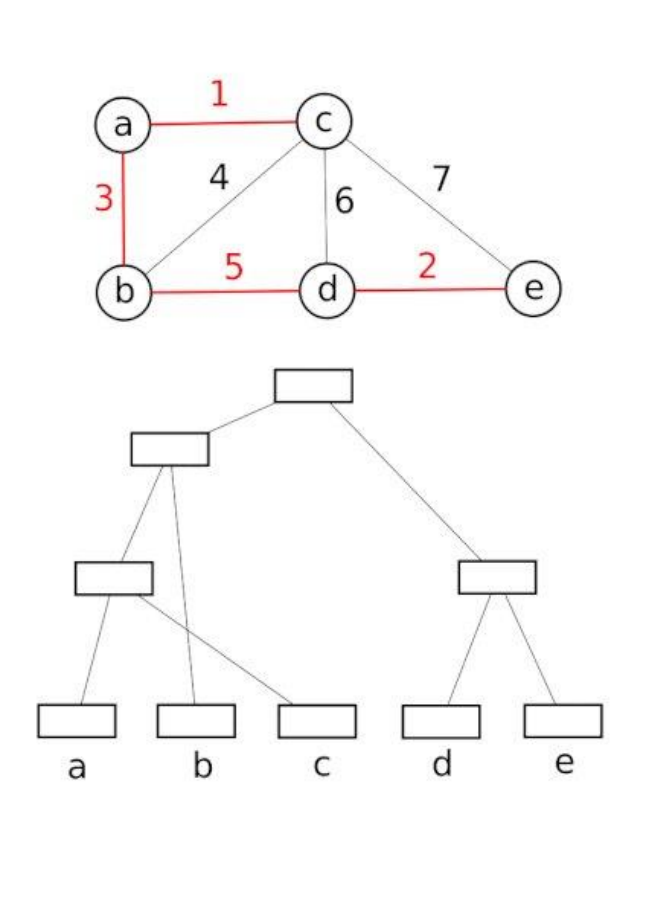
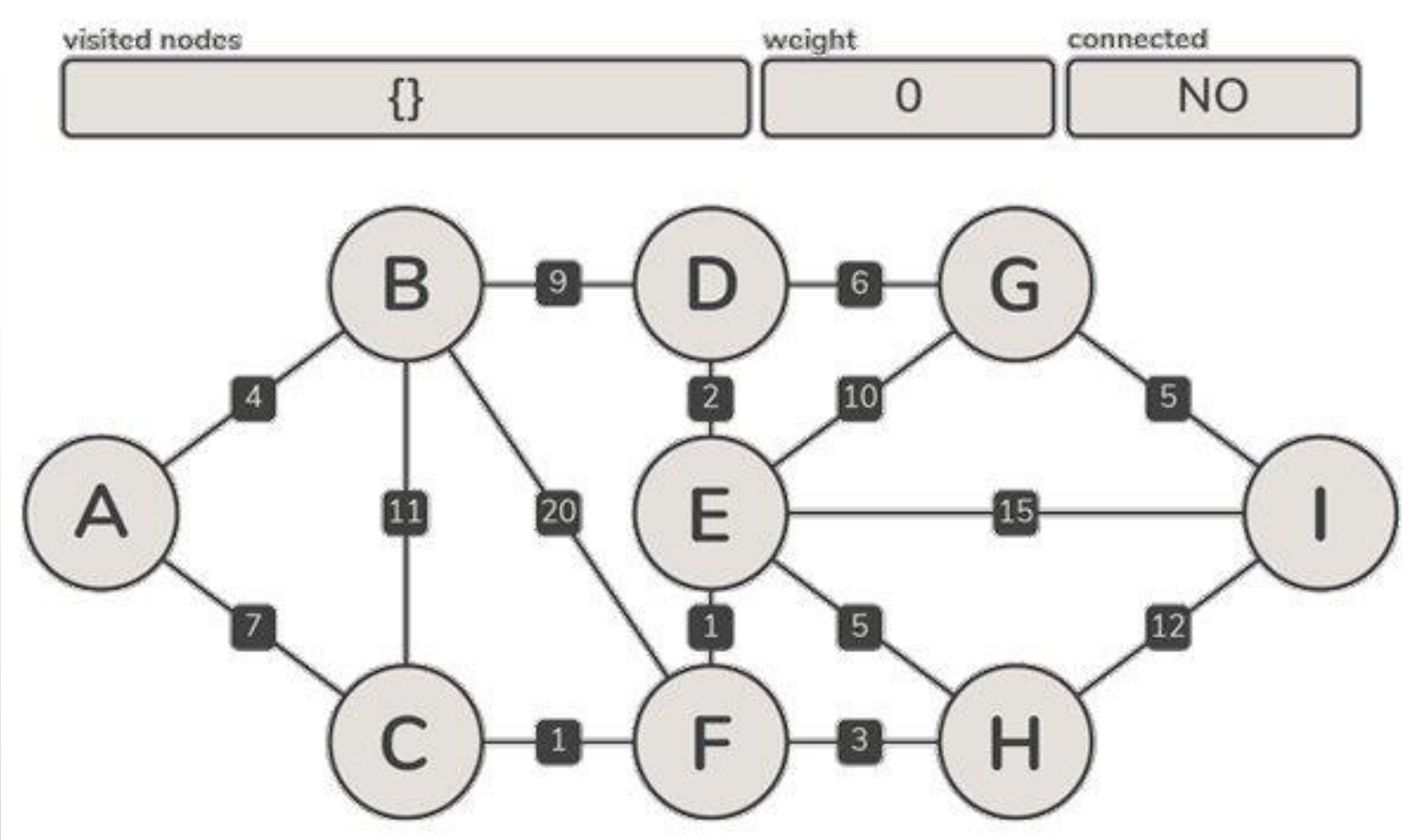
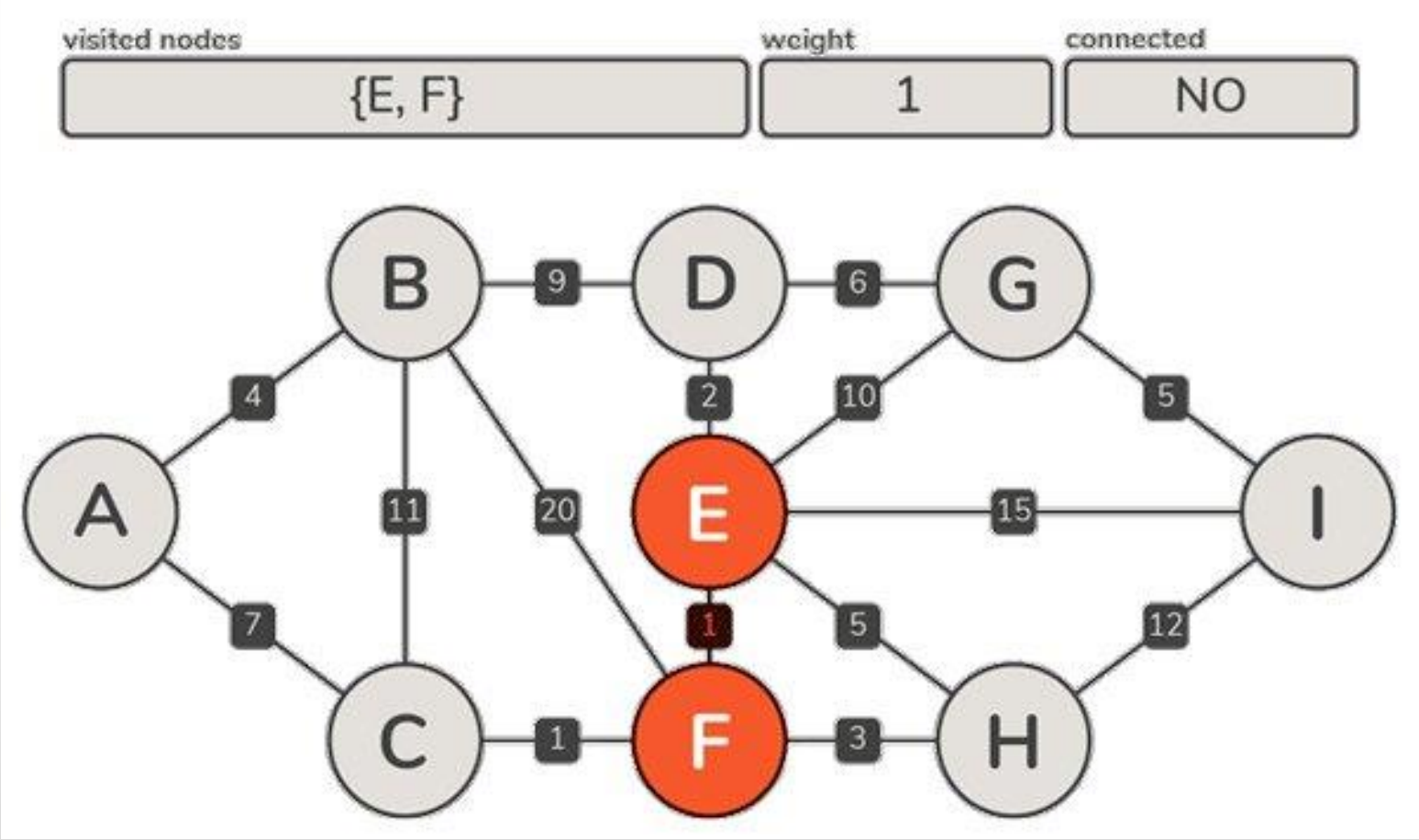


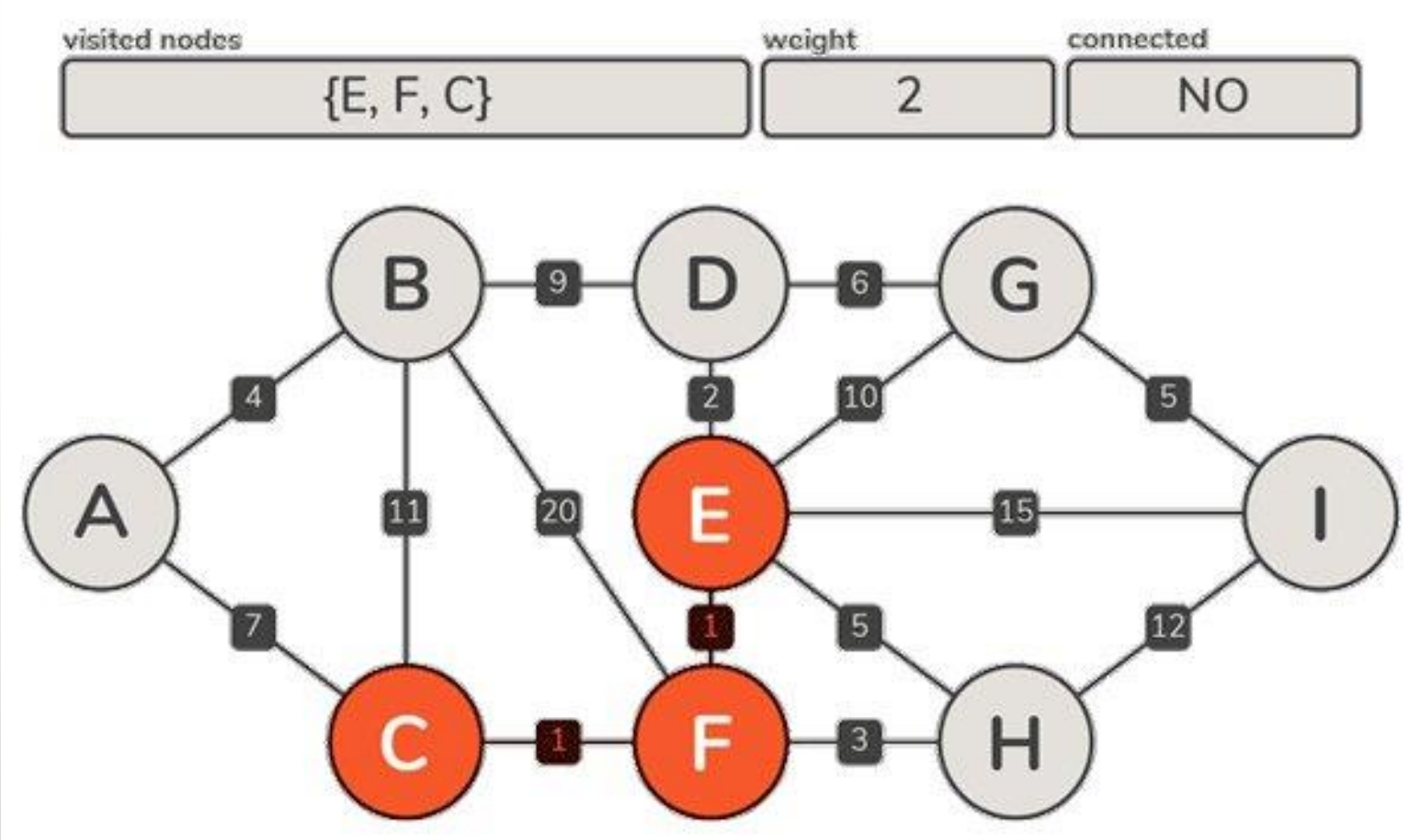
Örnek Kruskal



Örnek Kruskal



Örnek Kruskal



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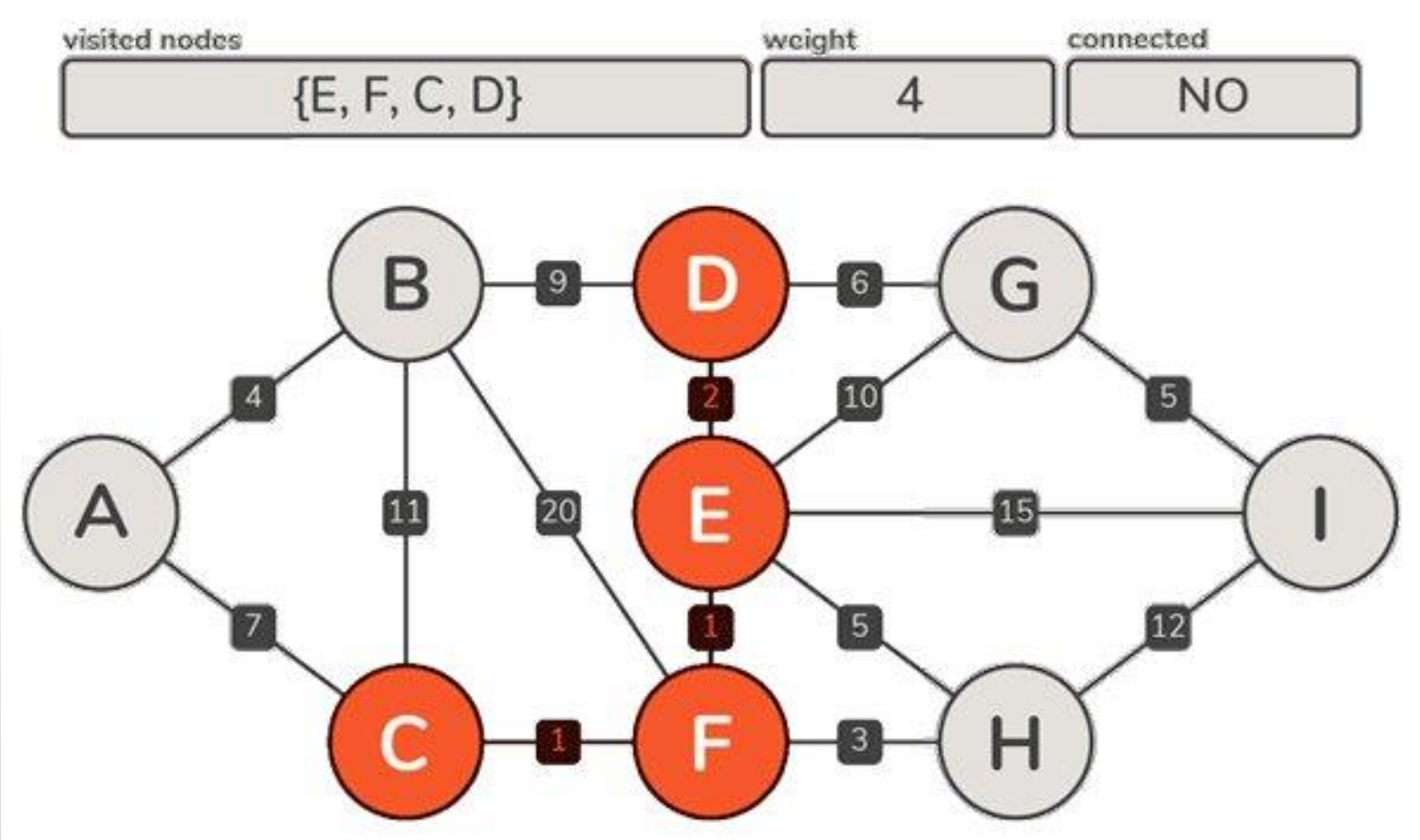

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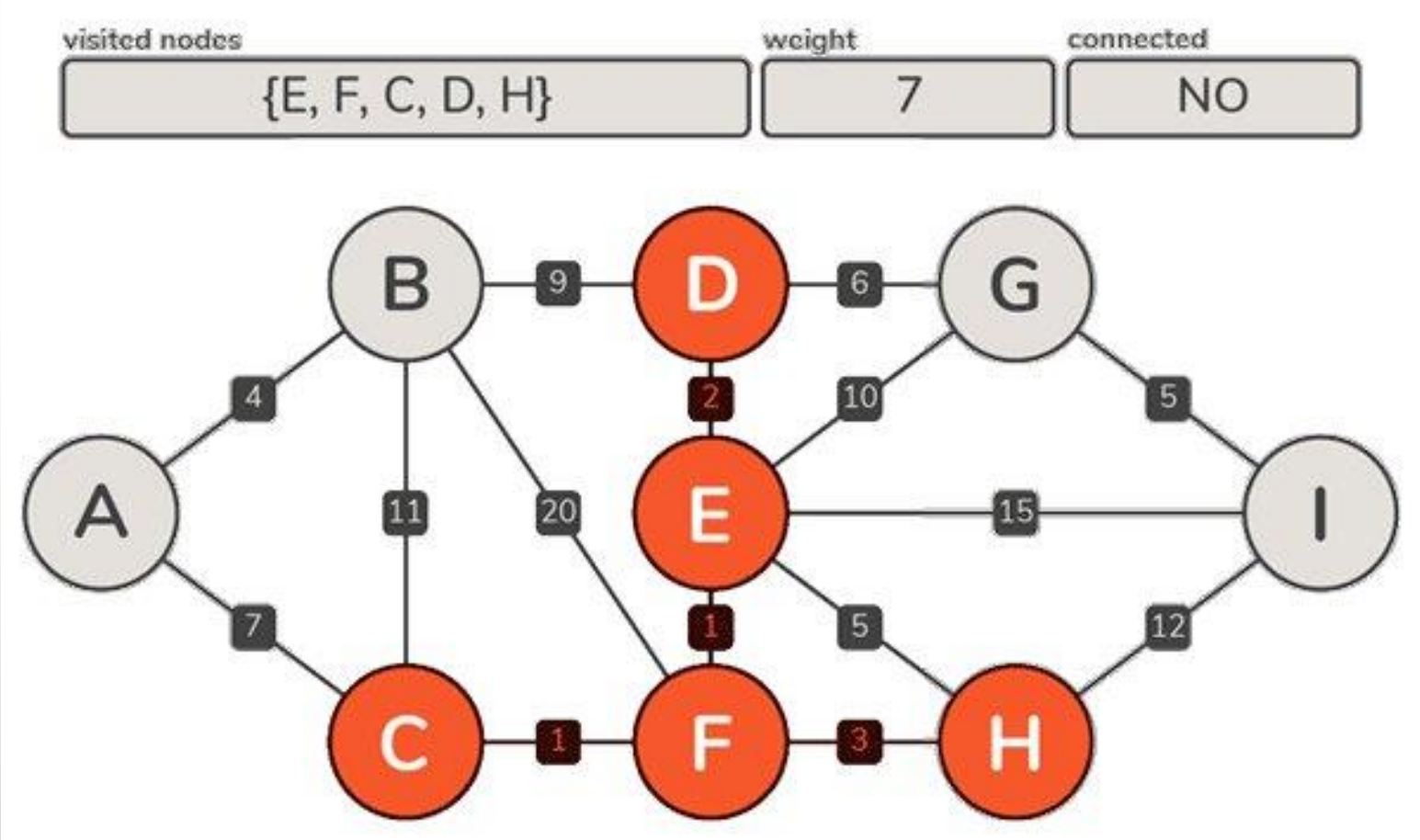

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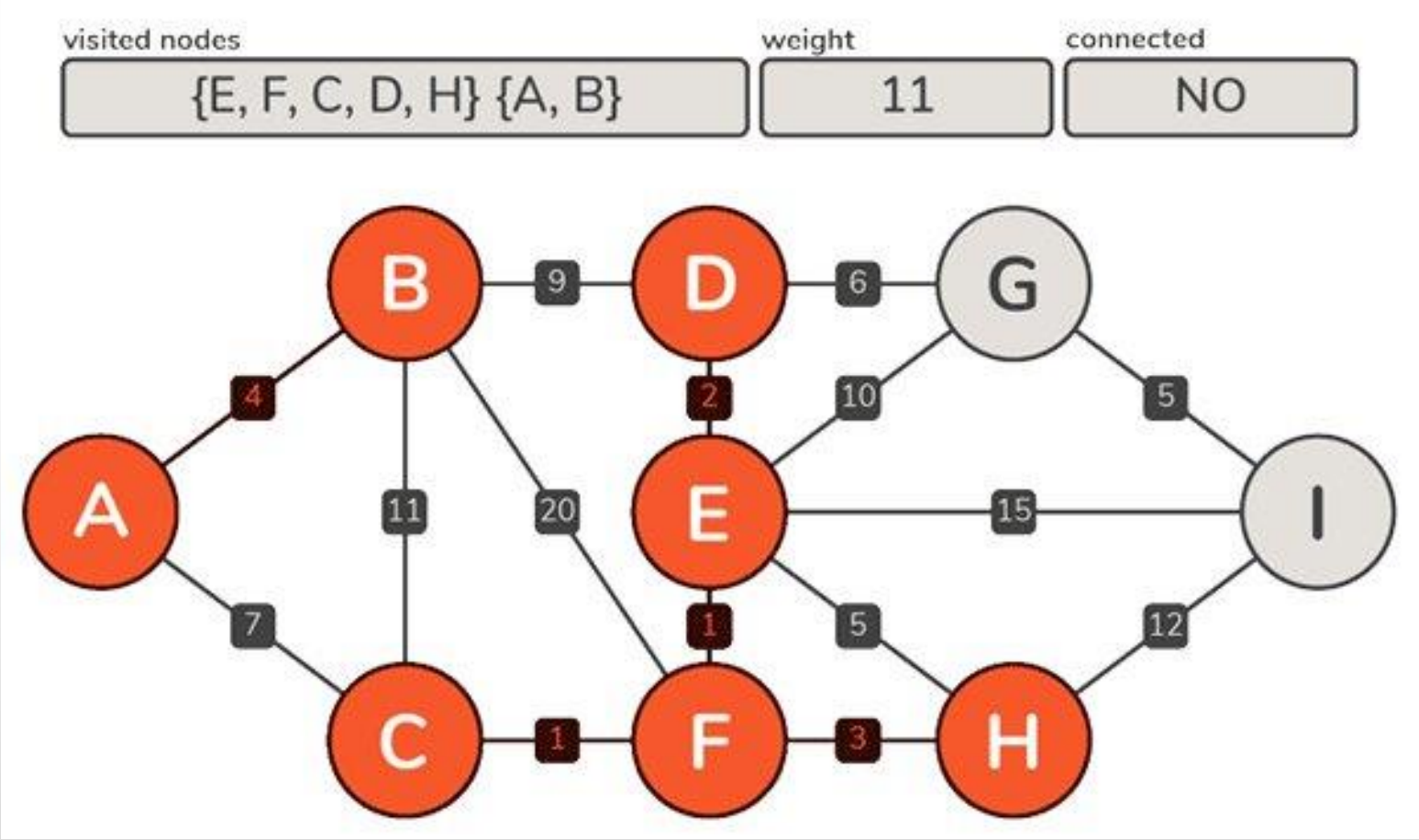

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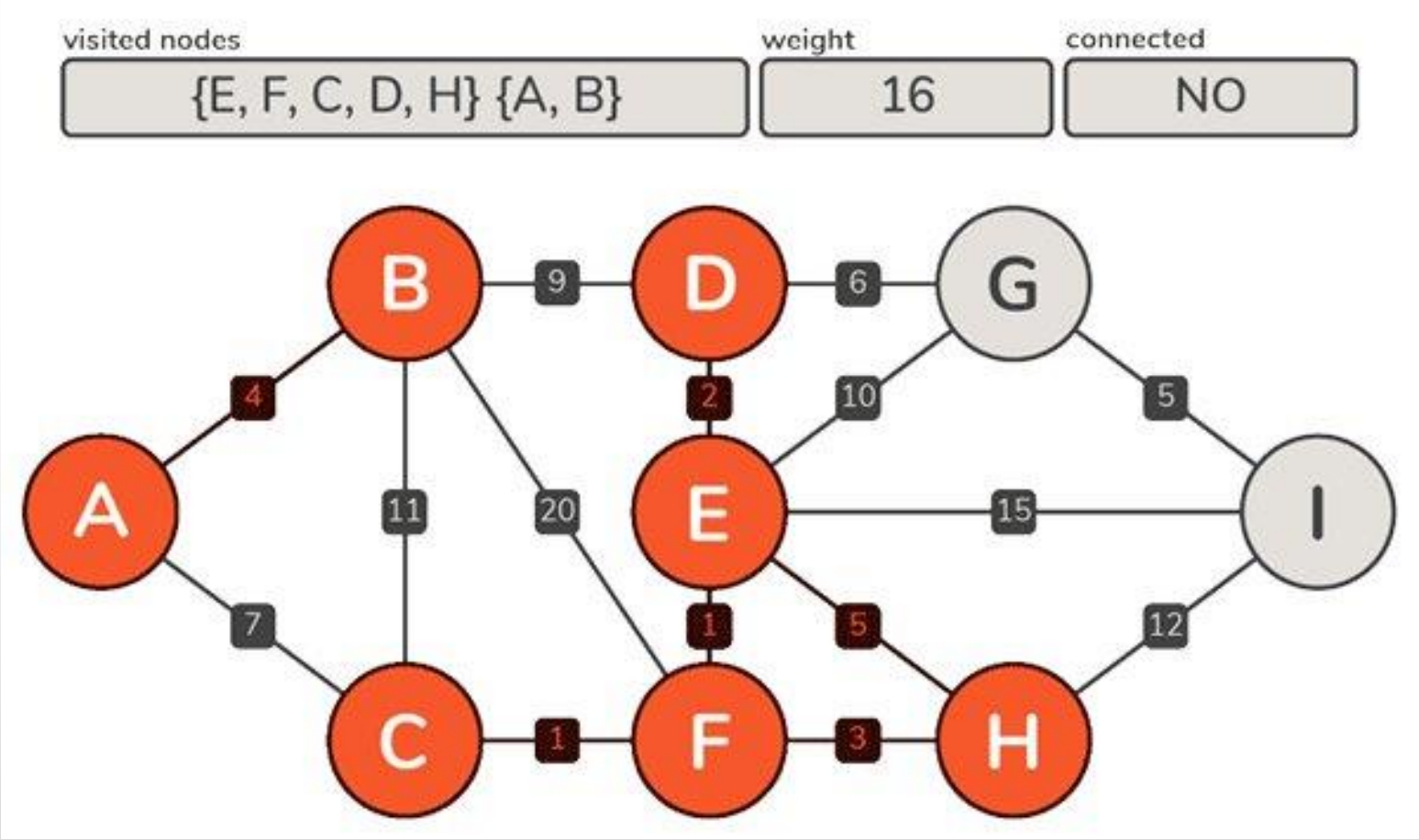

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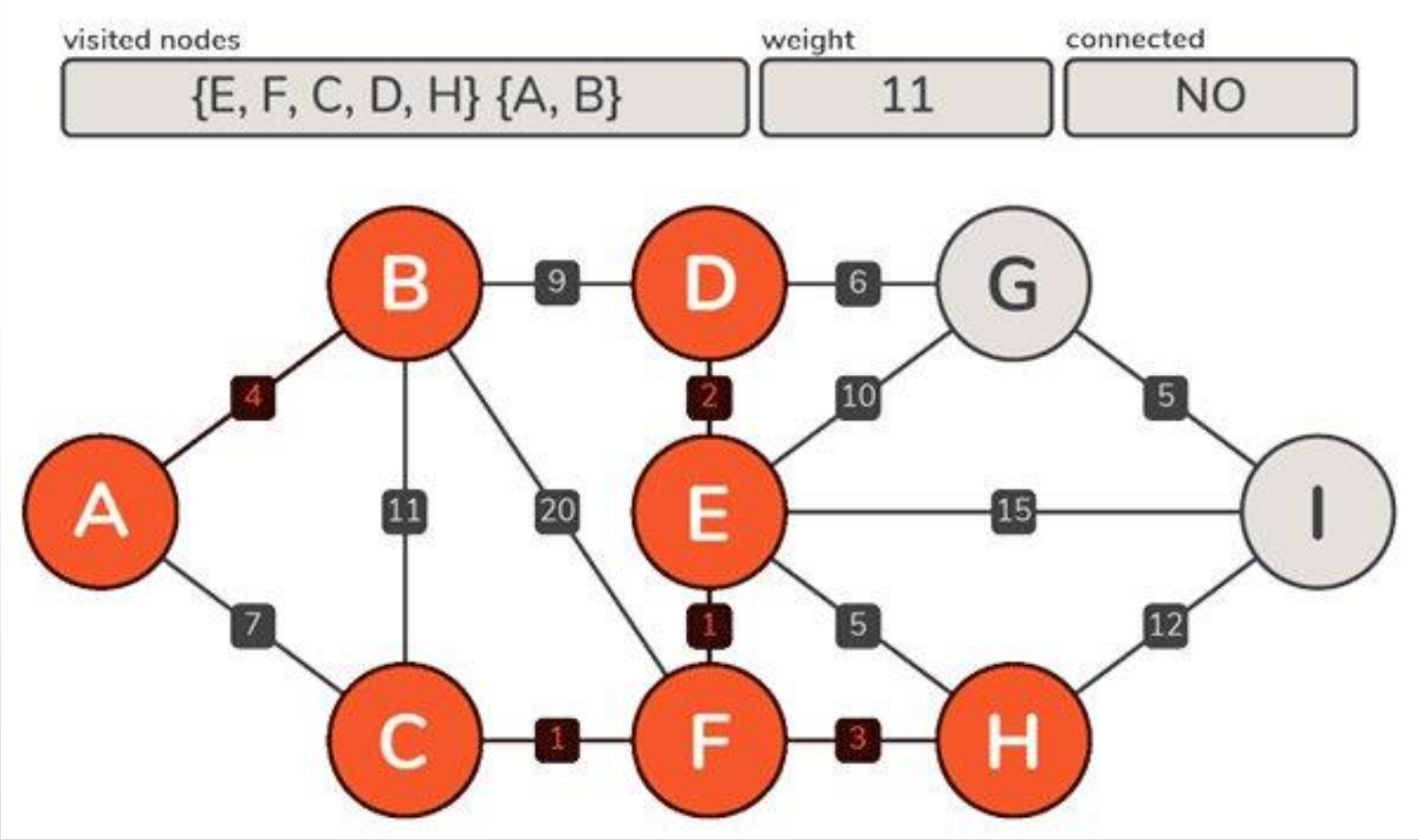

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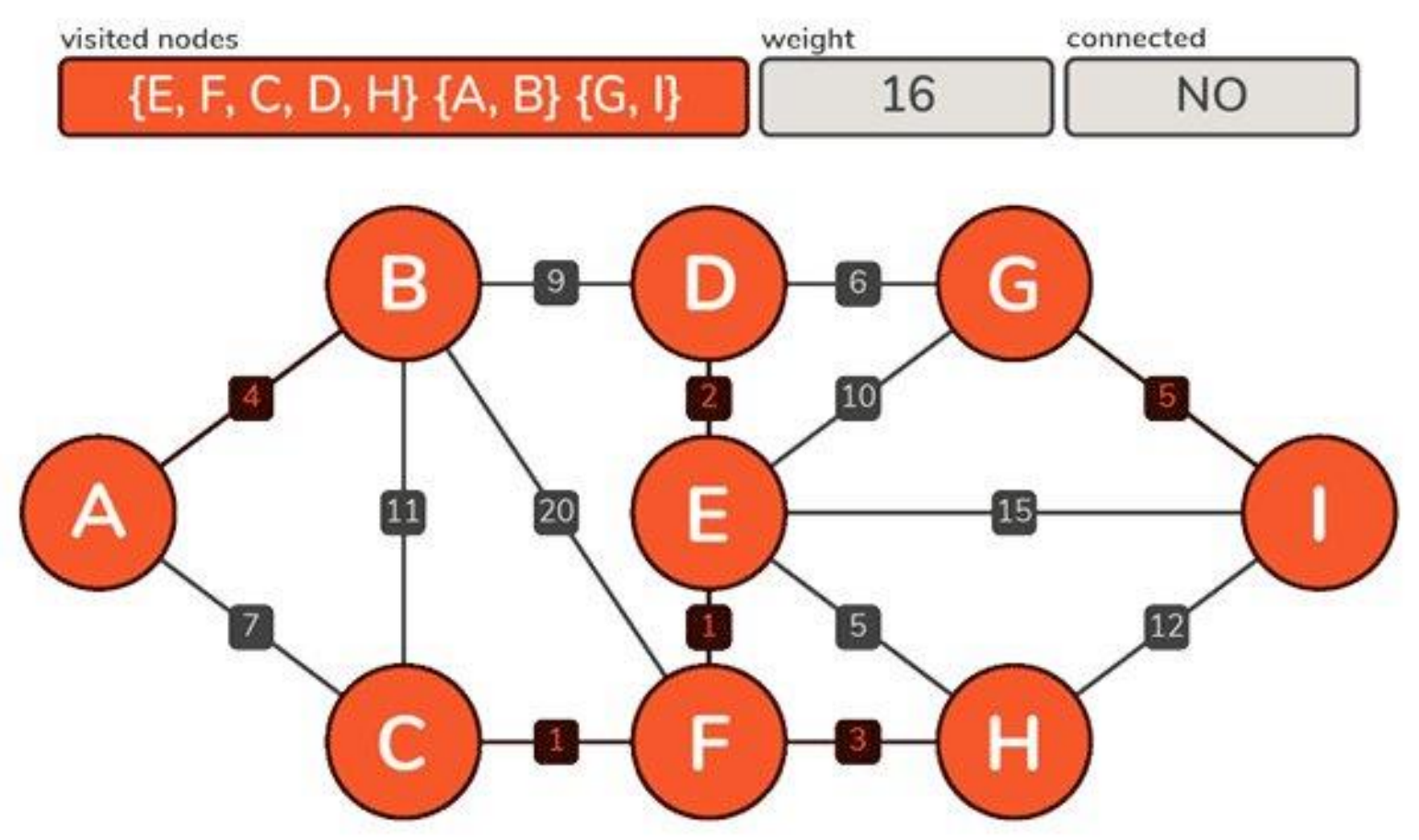

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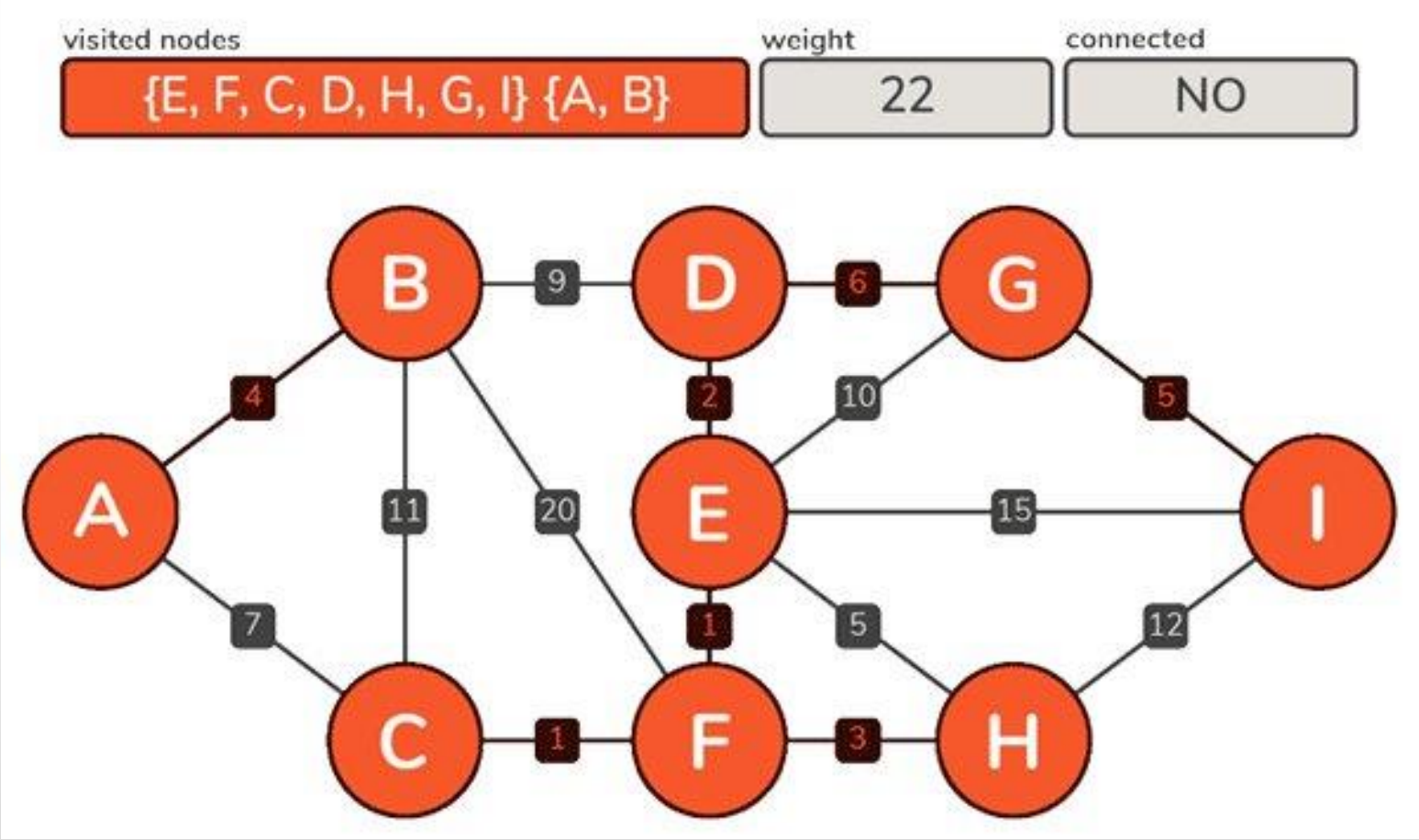

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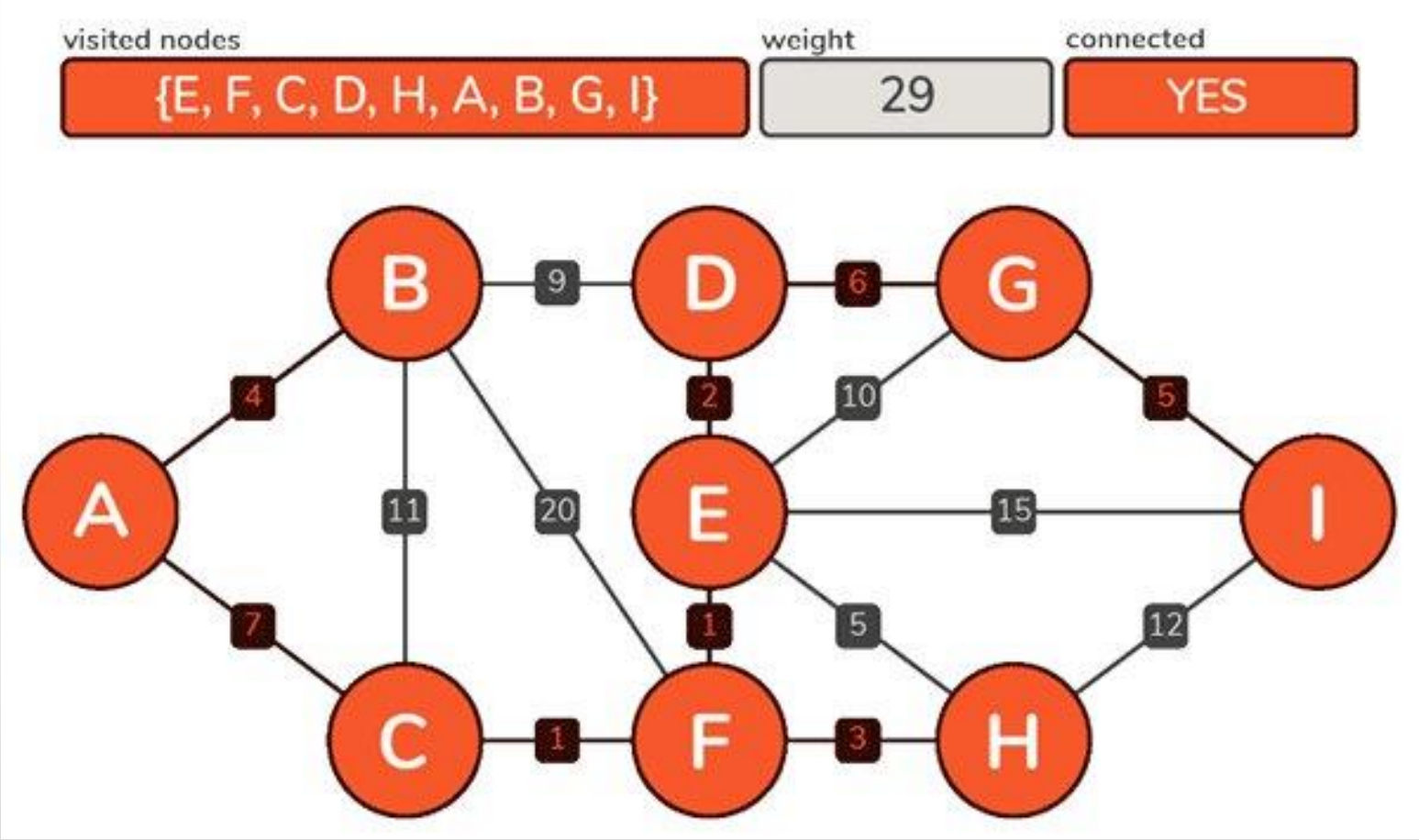

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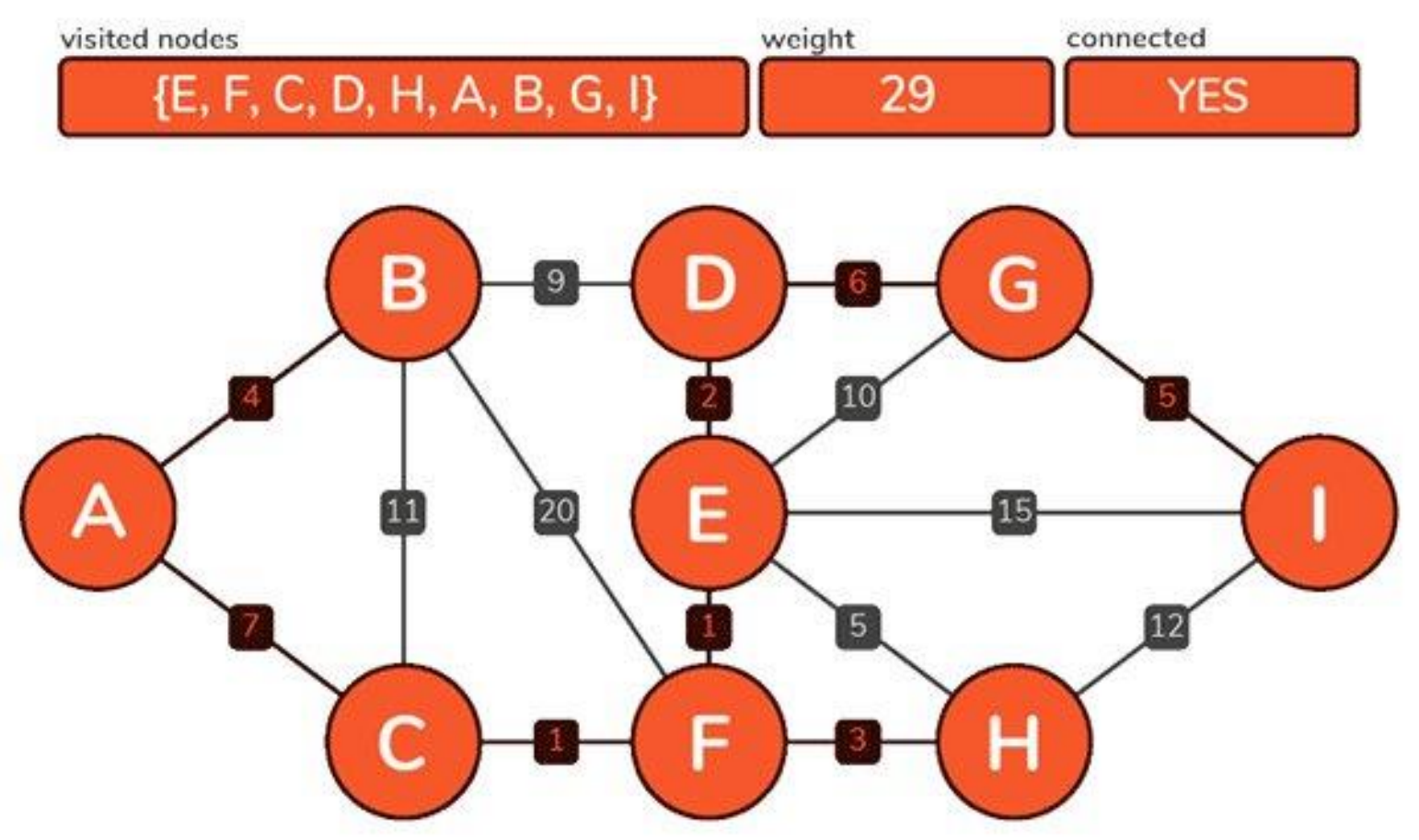


Prim Algoritması

- Tüm düğümleri bağlayan en küçük ağırlıklı ağacı oluşturmayı amaçlar.
- Açgözlü (greedy) bir algoritmadır.
- Robert C. Prim tarafından geliştirilmiştir.

Algoritma İlkeleri

- Ağırlıklı çizge üzerinde çalışır.
- Tüm düğümleri en küçük ağırlıklı kenarlarla birleştirir.
- Başlangıçta, bir düğüm seçilir ve ağacın başlangıç düğümü kabul edilir.
- Her adımda,
  - ağaç içinde olmayan düğümler arasından,
  - ağaçta olan en küçük ağırlıklı kenar ile yeni bir düğüm eklenir.

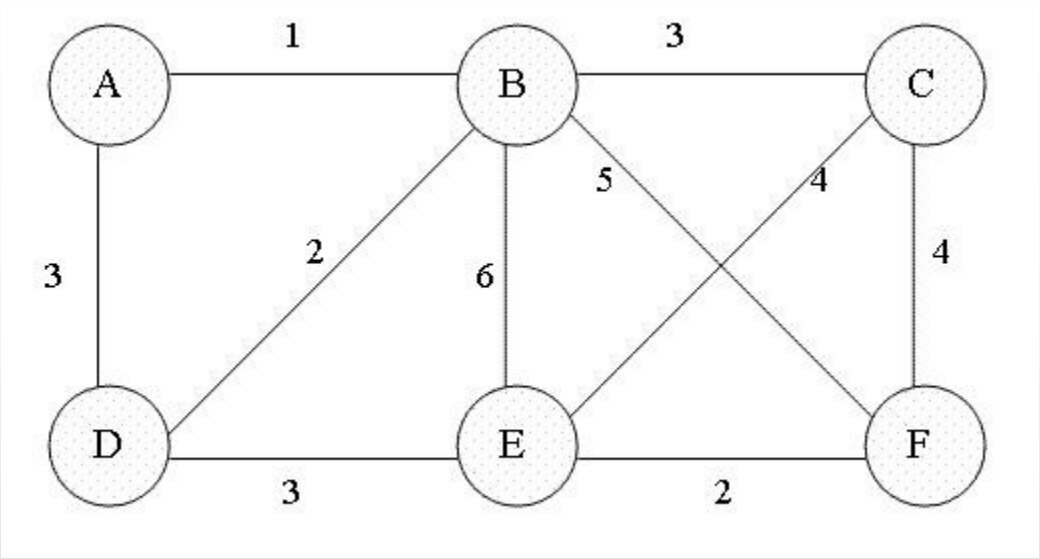
Algoritma Adımları

- Adım 1: Başlangıç düğümü seçilir ve bu düğüme ait olan tüm kenarlar bir öncelik kuyruğuna eklenir.
- Adım 2: Kuyruktan, ağaçta olmayan en küçük ağırlıklı kenar seçilir.
- Adım 3: Seçilen kenar ile bağlantılı olan yeni düğüm ağaca eklenir.
- Adım 4: Yeni eklenen düğüme bağlı kenarlar öncelik kuyruğuna eklenir.

Karmaşıklık Analizi

- Prim Algoritması'nın karmaşıklığı
  - *O(E + V log V)* veya
  - *O(E log V)* olarak ifade edilir.
- E kenar sayısını,
- V düğüm sayısını temsil eder.

Prim



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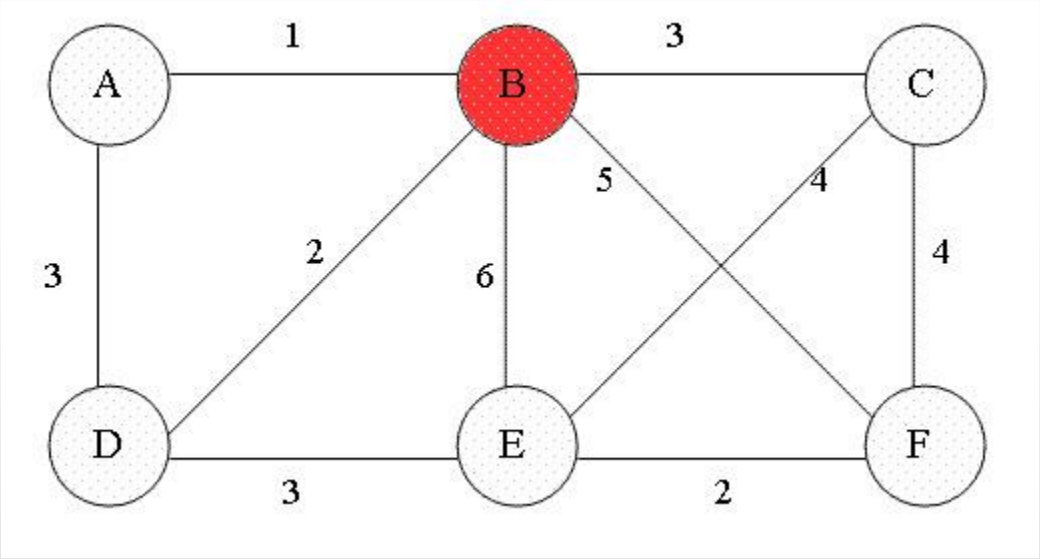

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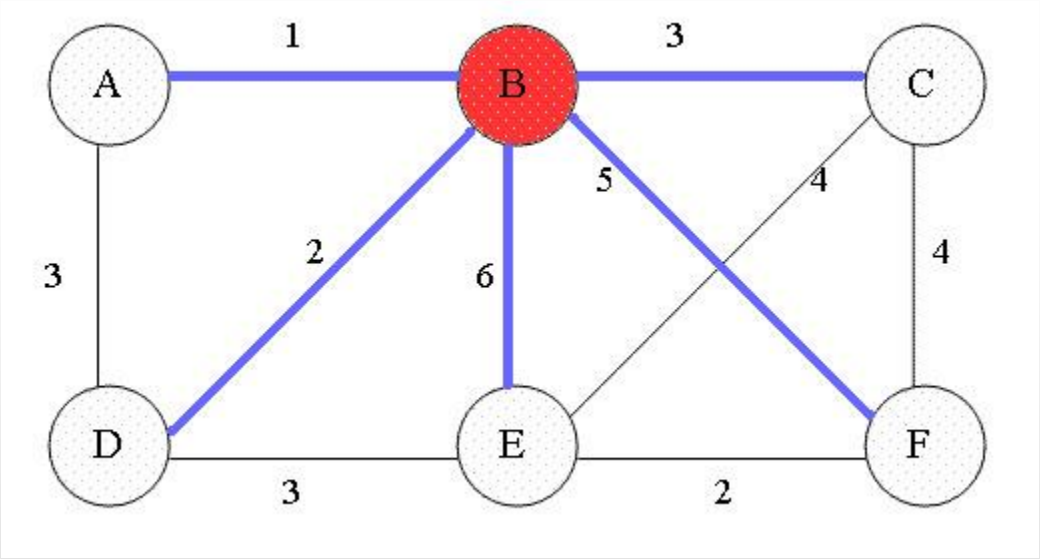

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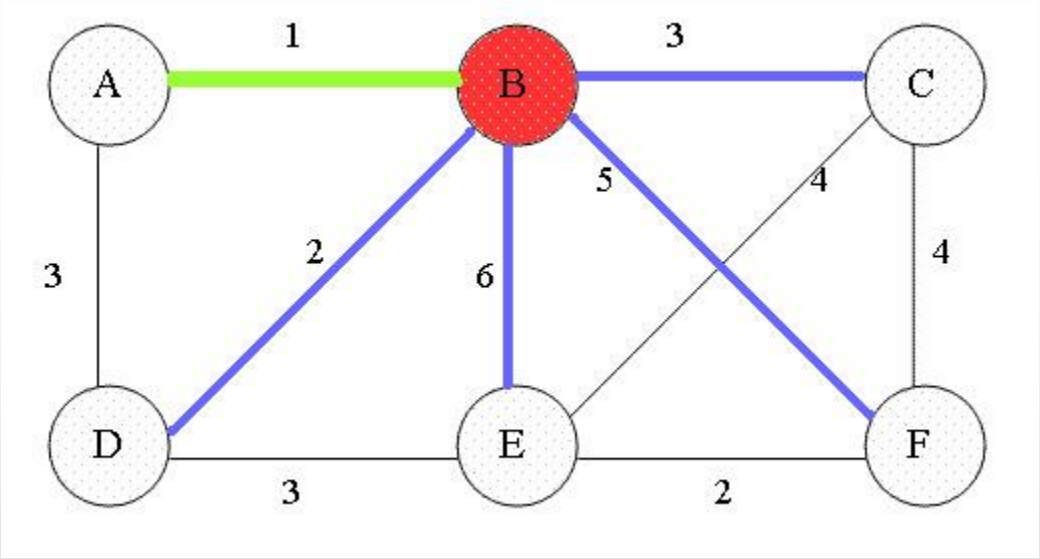

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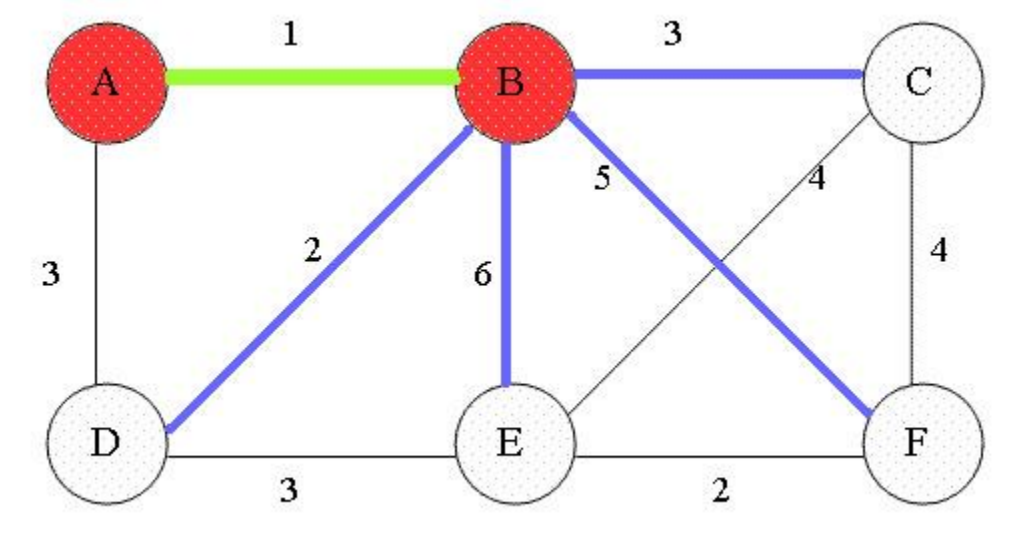

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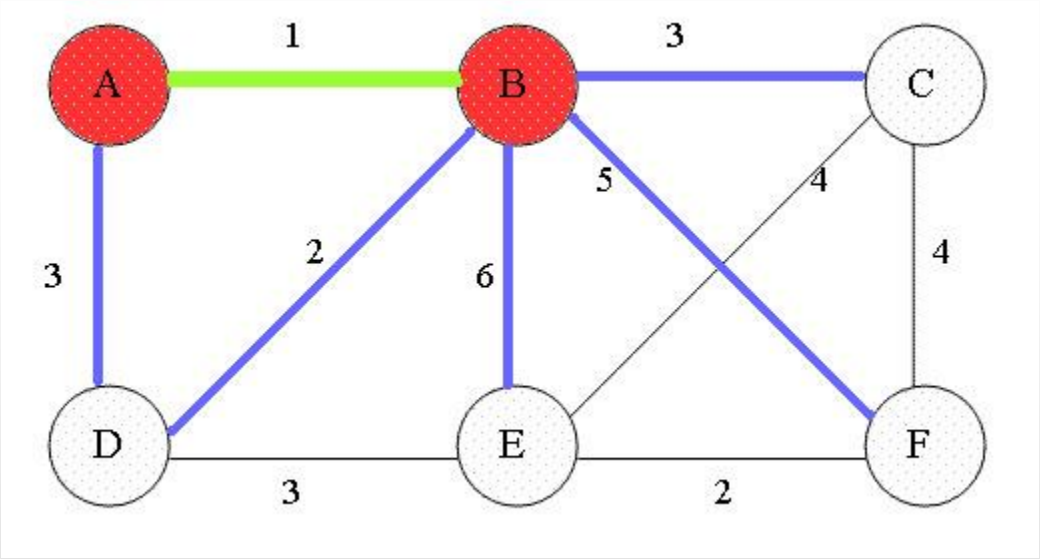

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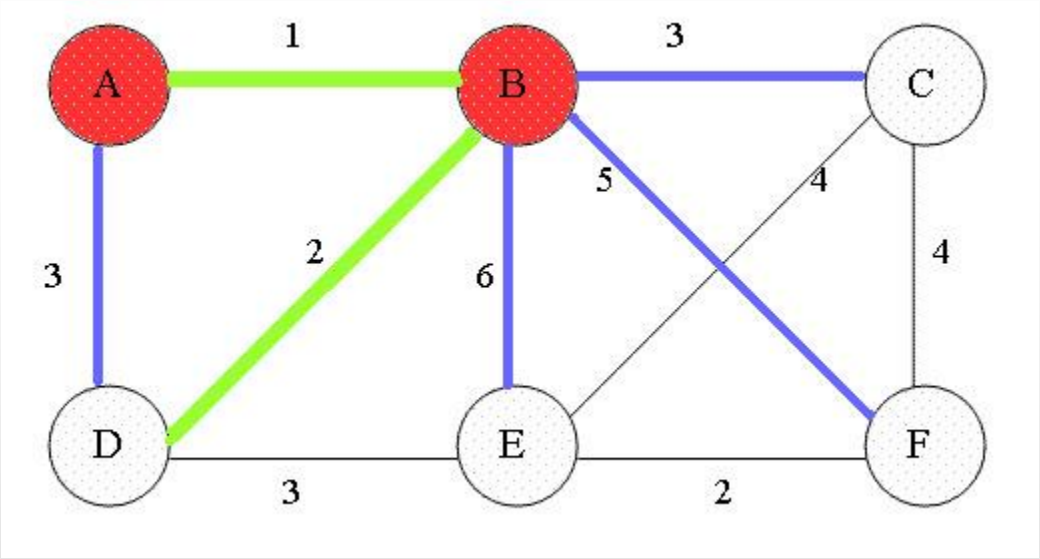

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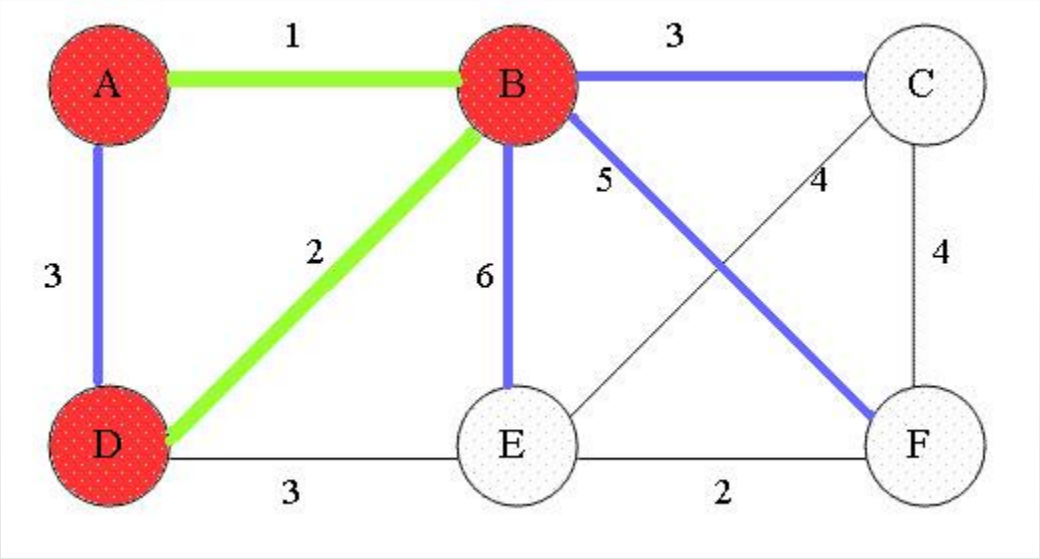

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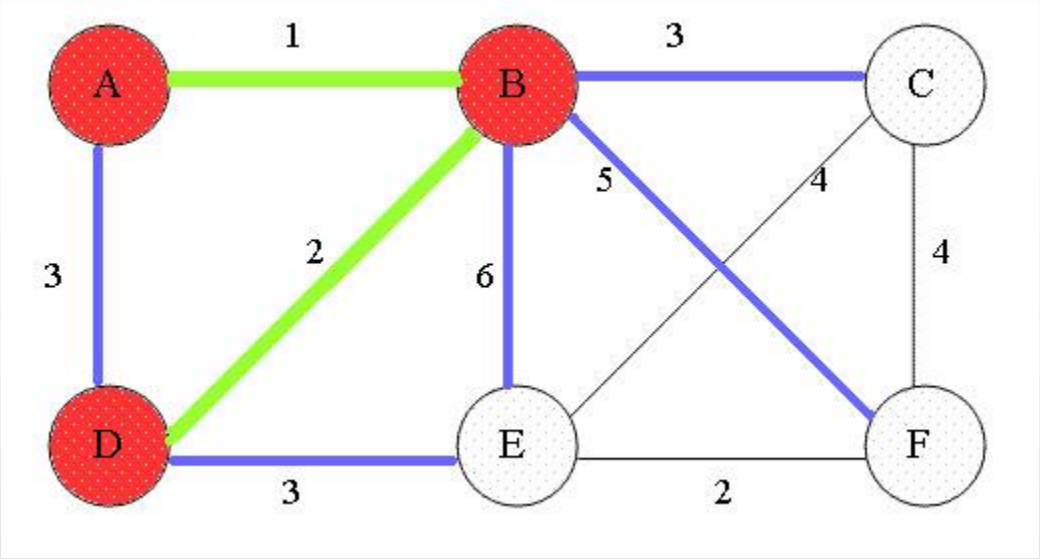

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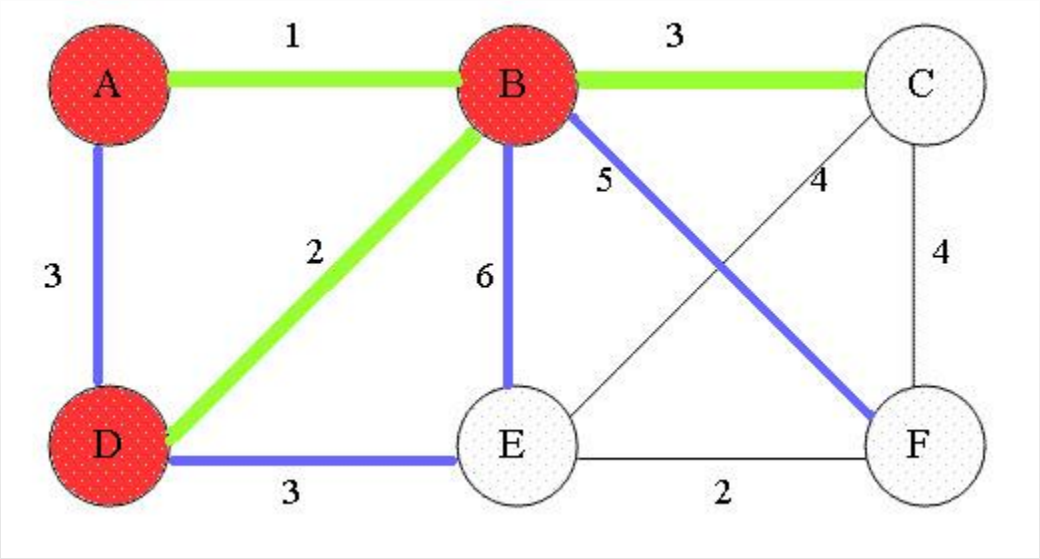

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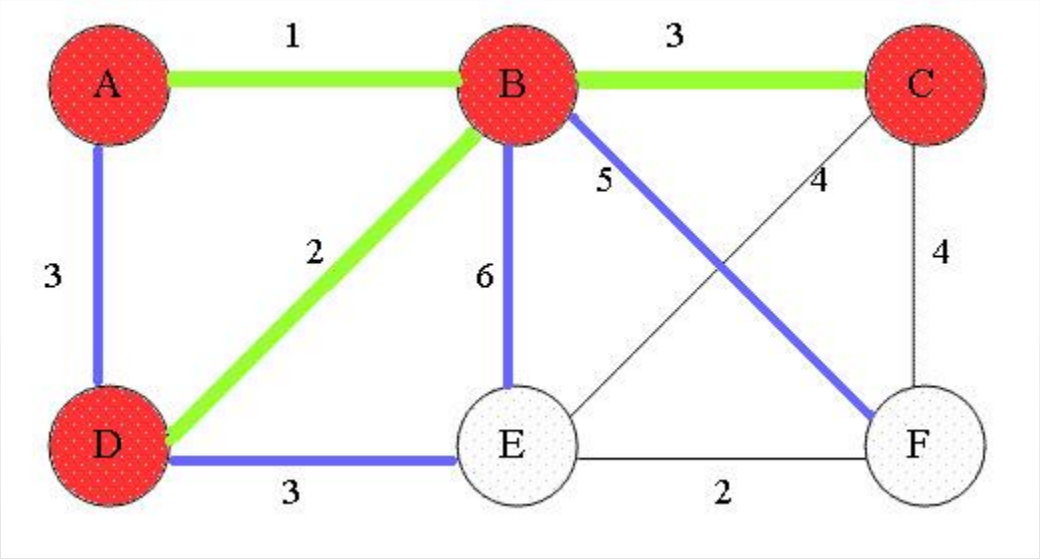

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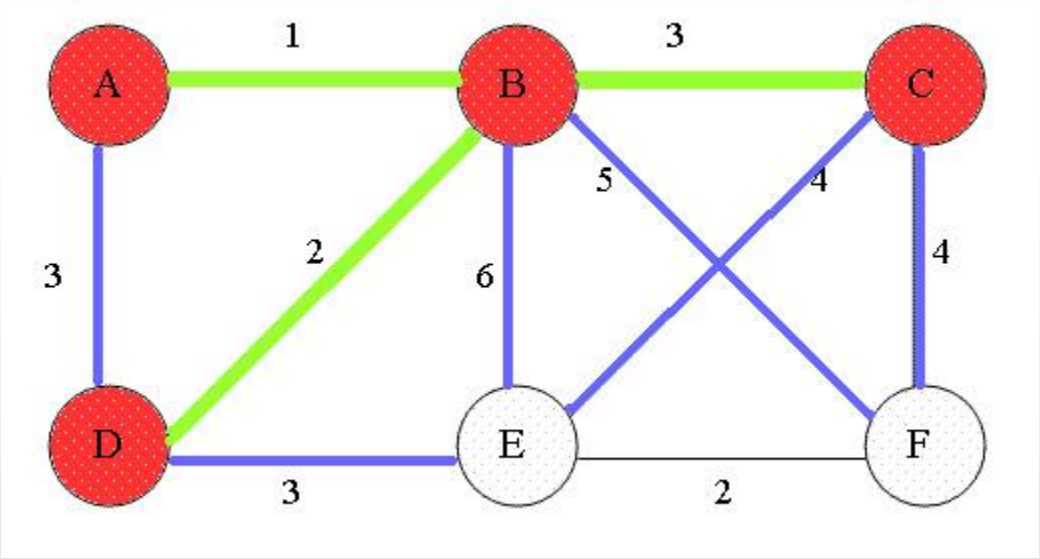

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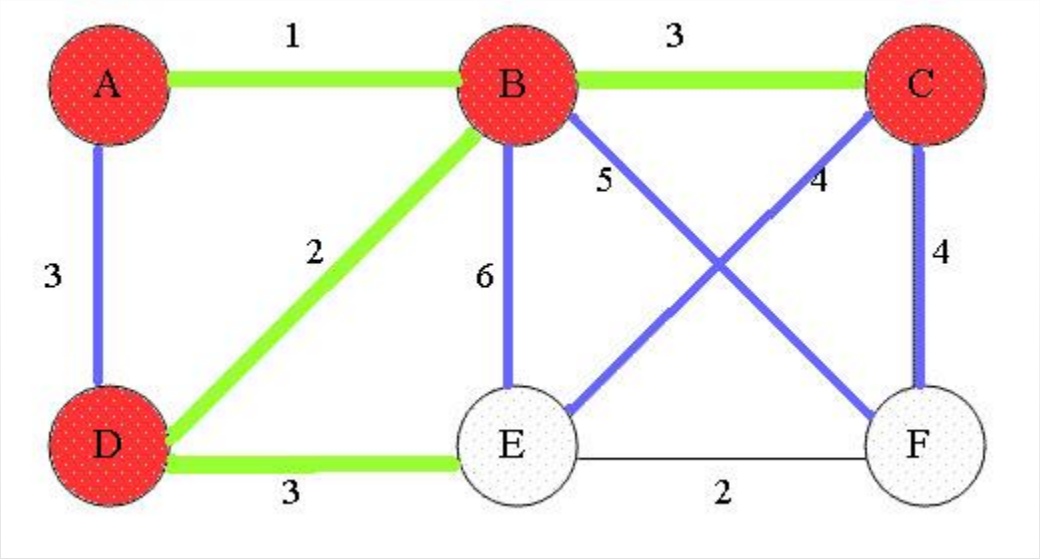

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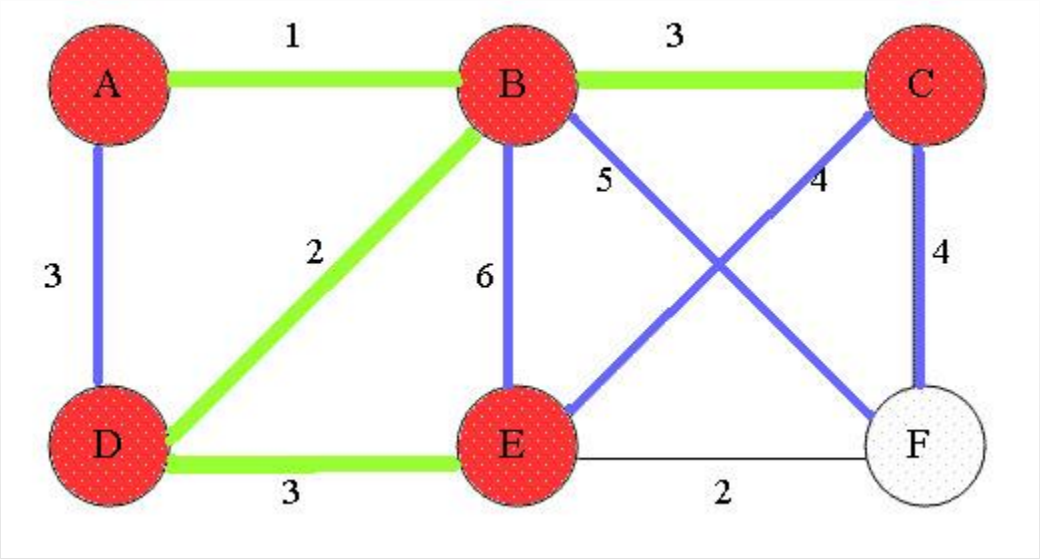

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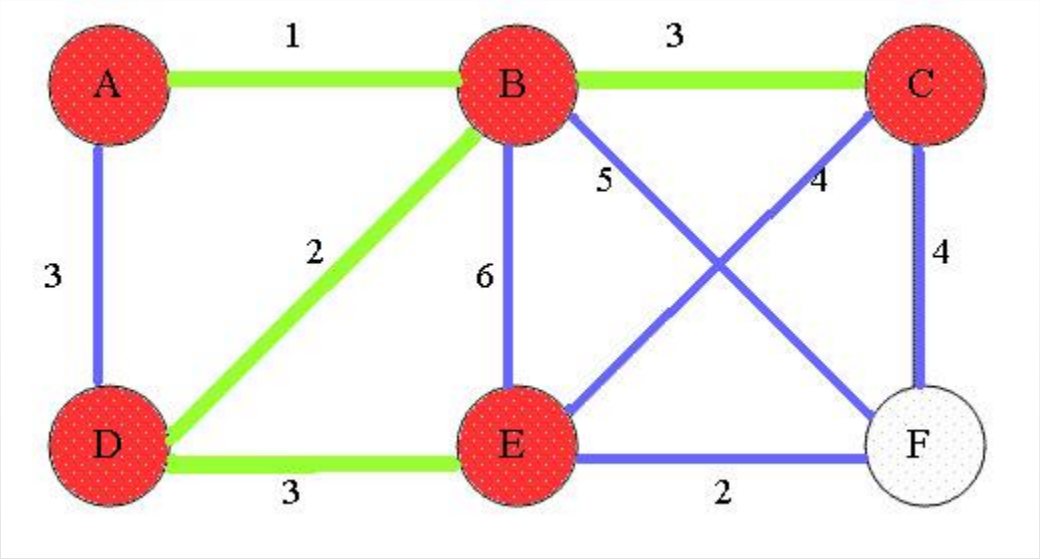

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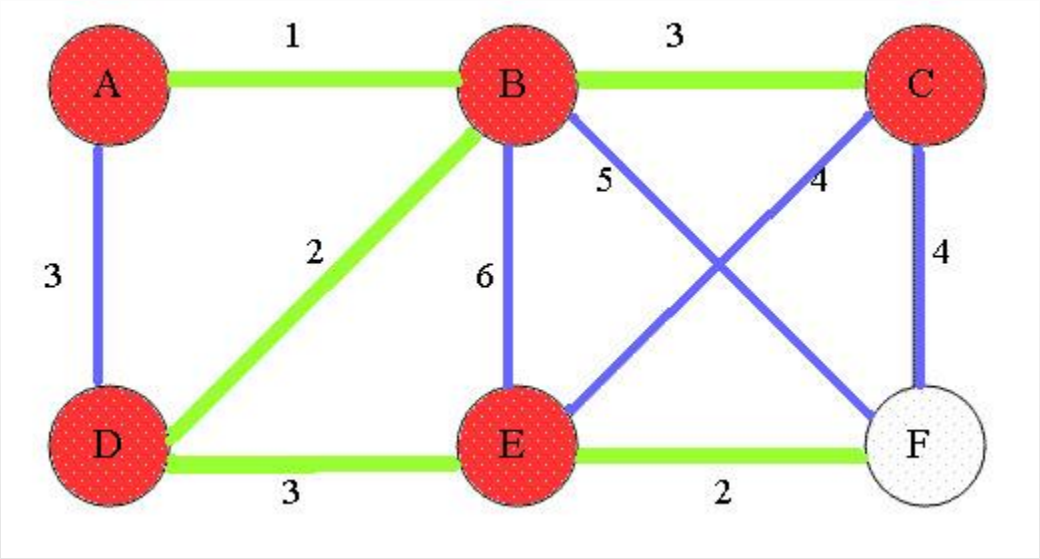

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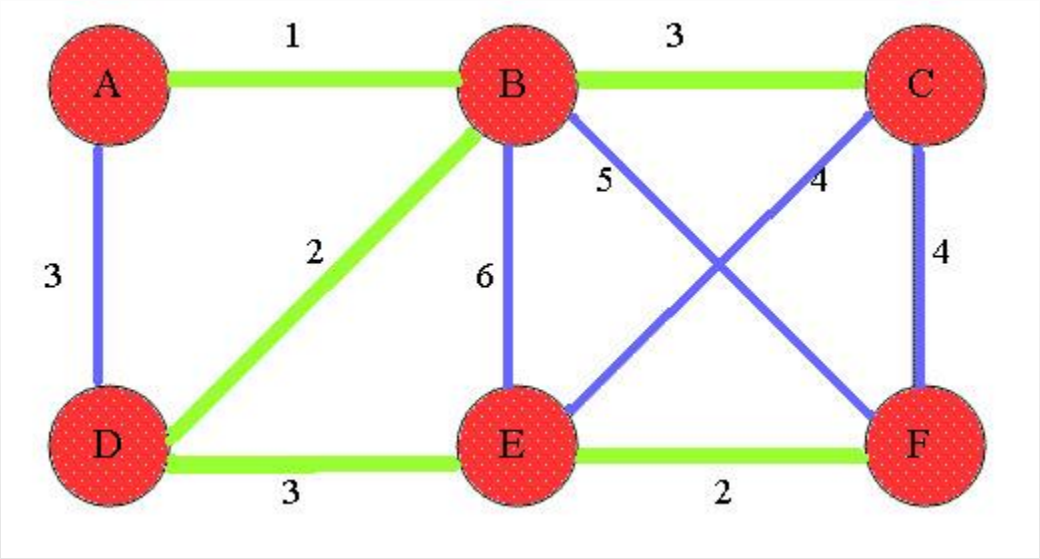

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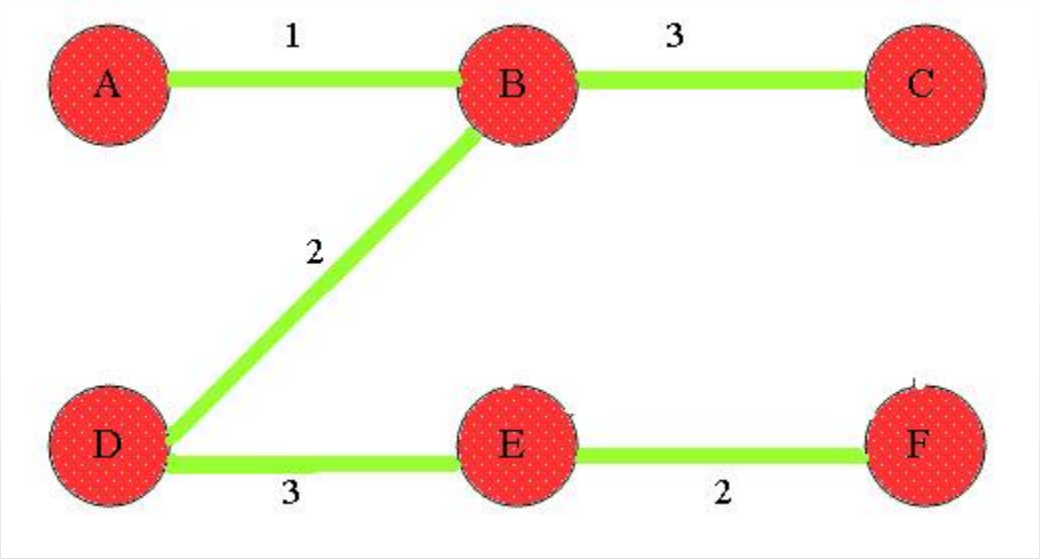
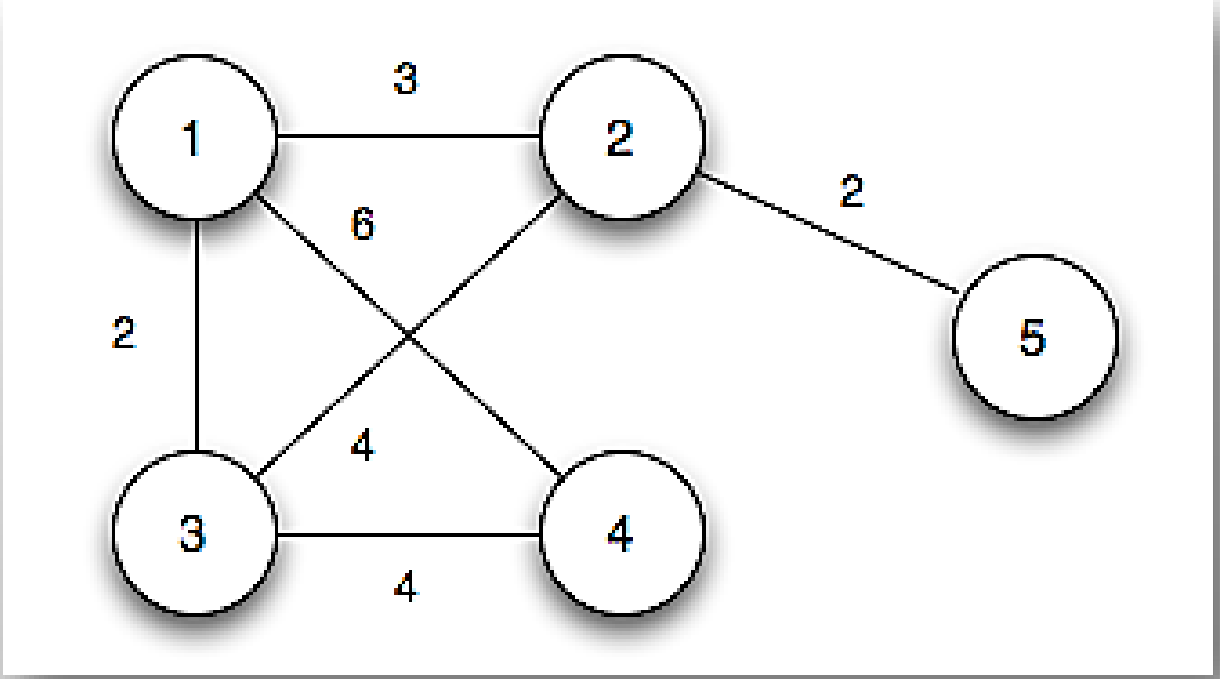


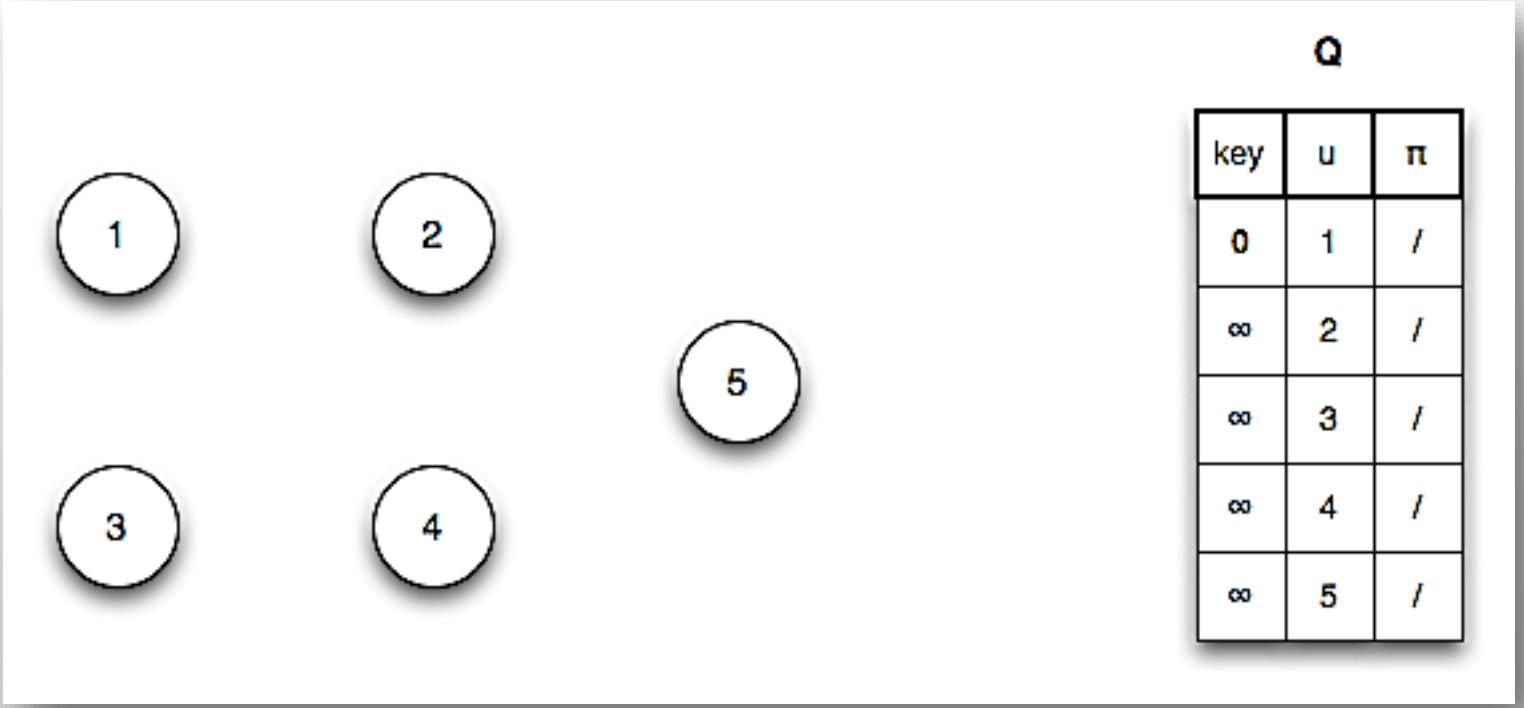
Örnek

- Aşağıdaki yönsüz çizge verilsin.



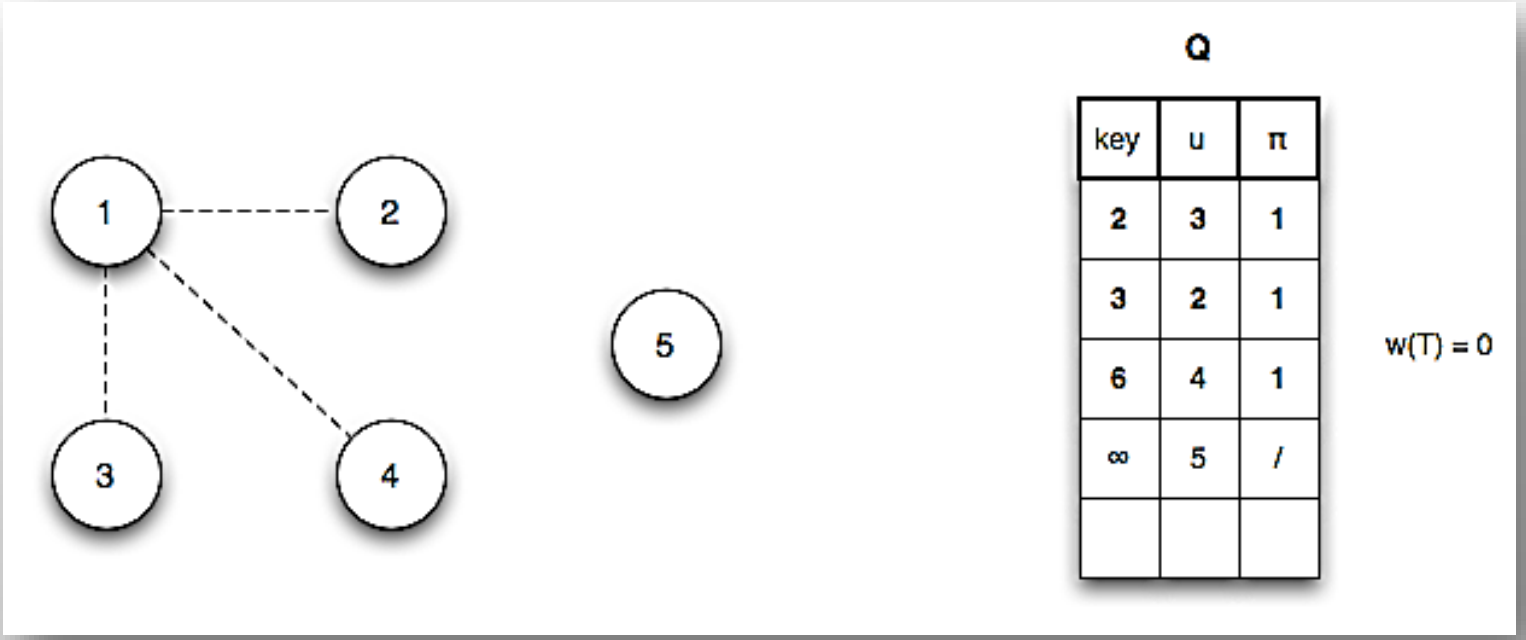
İlklendirme Aşaması

- Düğüm 1 ile kuyruk başlatılır.



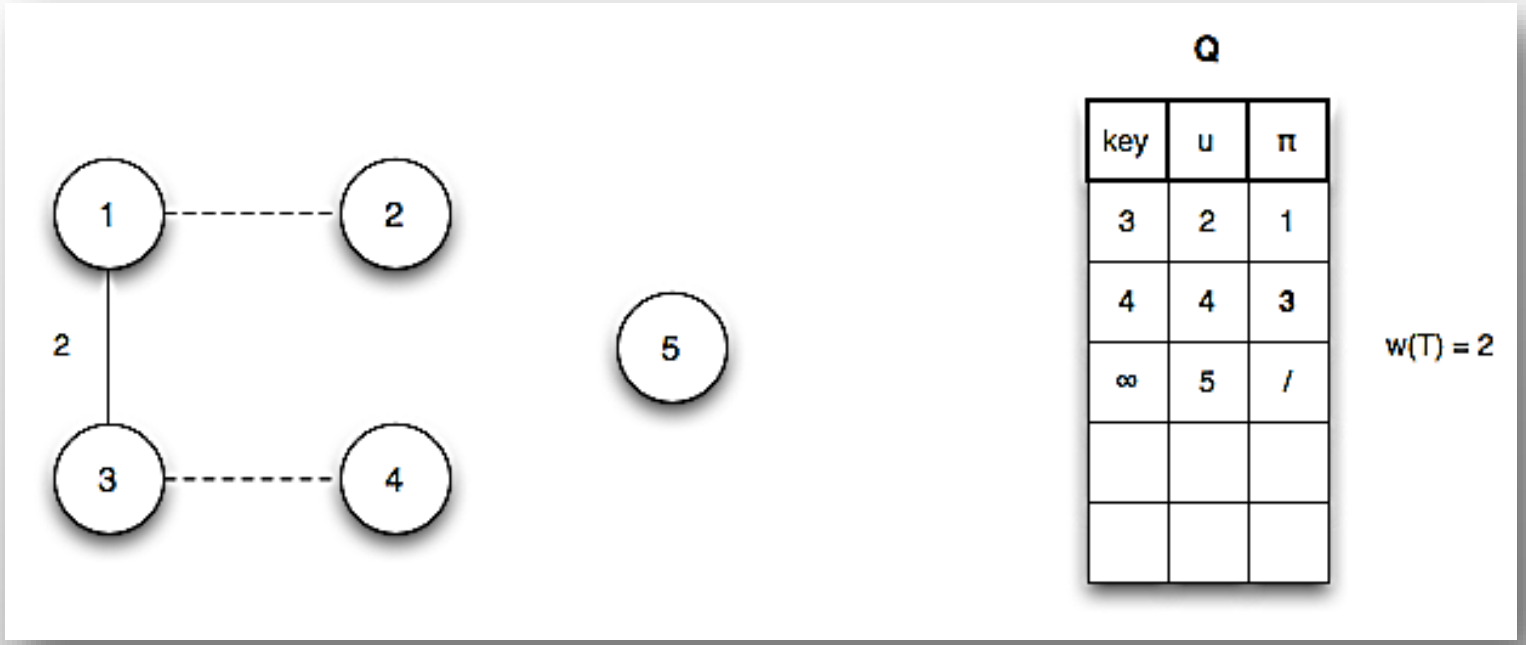
Adım 1

- Kuyruktan düğüm 1'i al, Q'yu güncelle.
- u3.key = 2 ((u1,u3)), u2.key = 3 ((u1,u2)), u4.key = 6 ((u1,u4))



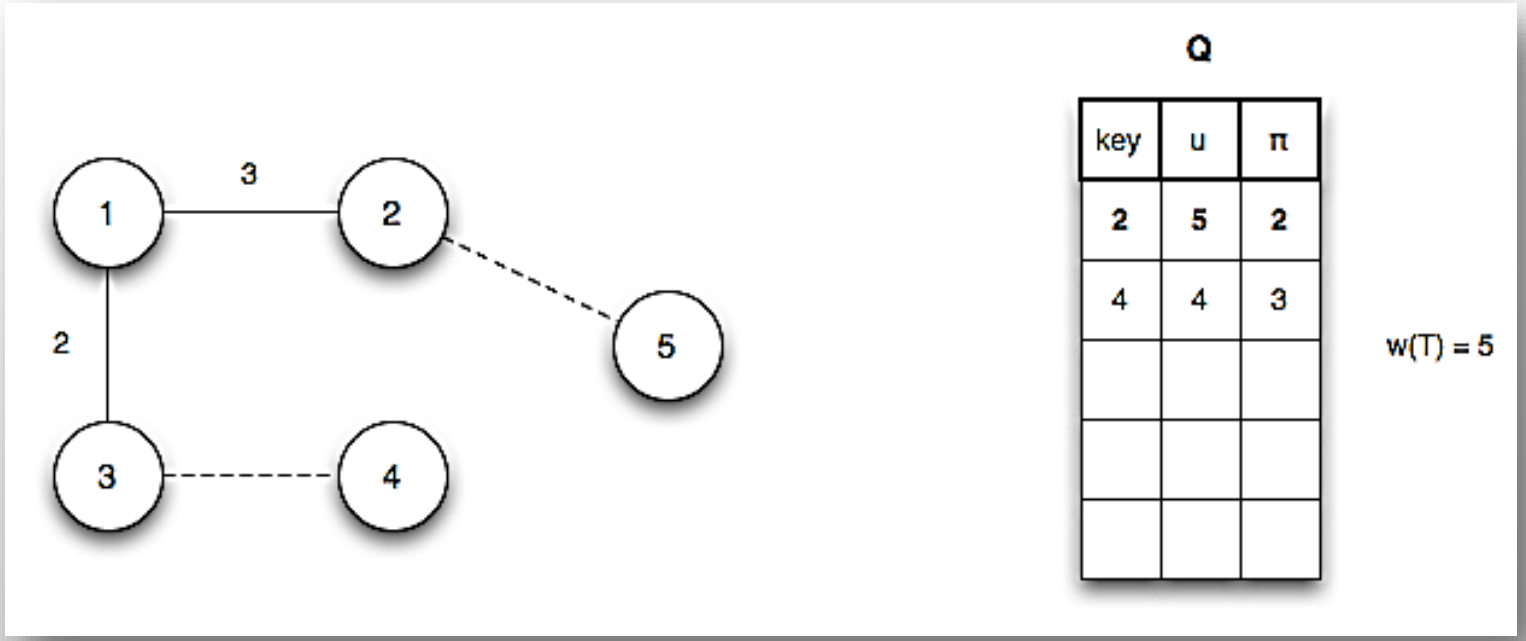
Adım 2

- Kuyruktan düğüm 3'ü al, T'ye (u_1,u_3) kenarını ekle. Q'yu güncelle.
- u4.key = 4 ((u3,u4))



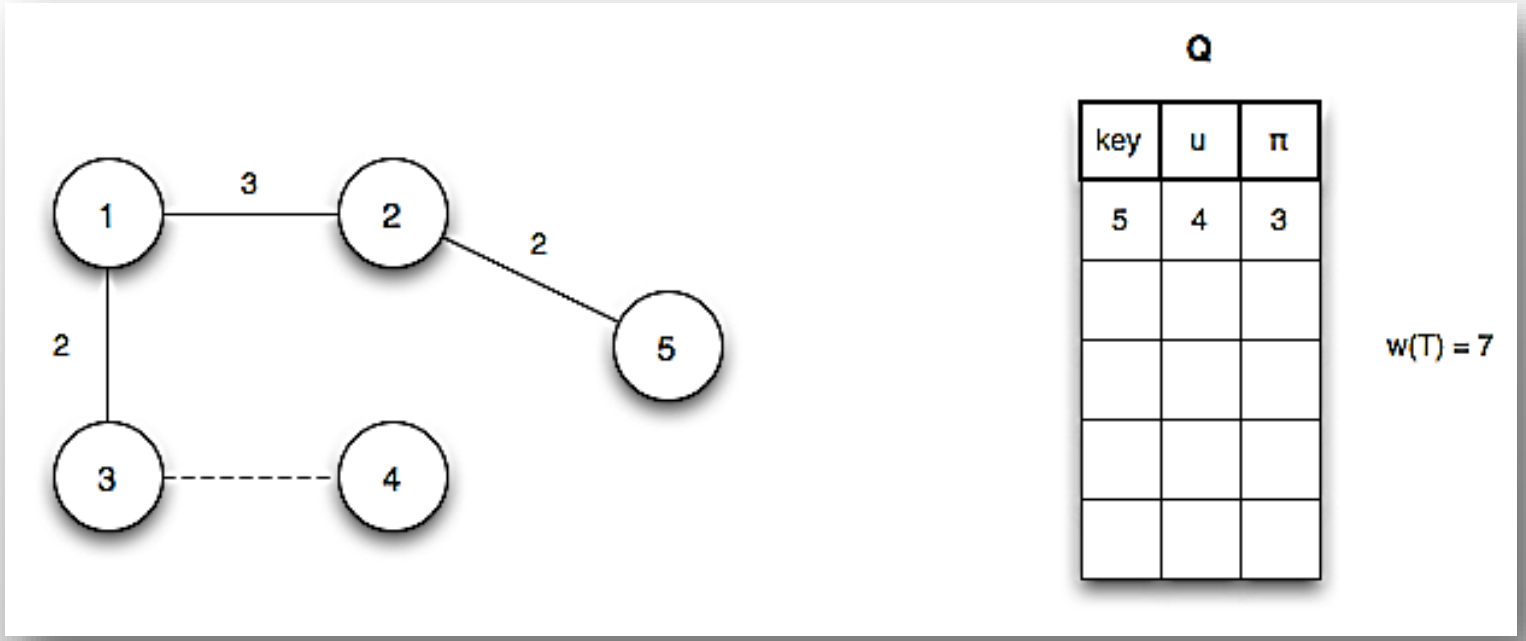
Adım 3

- Kuyruktan düğüm 2'yi al, T'ye (u1,u2) kenarını ekle. Q'yu güncelle.
- u5.key = 2 ((u2,u5))



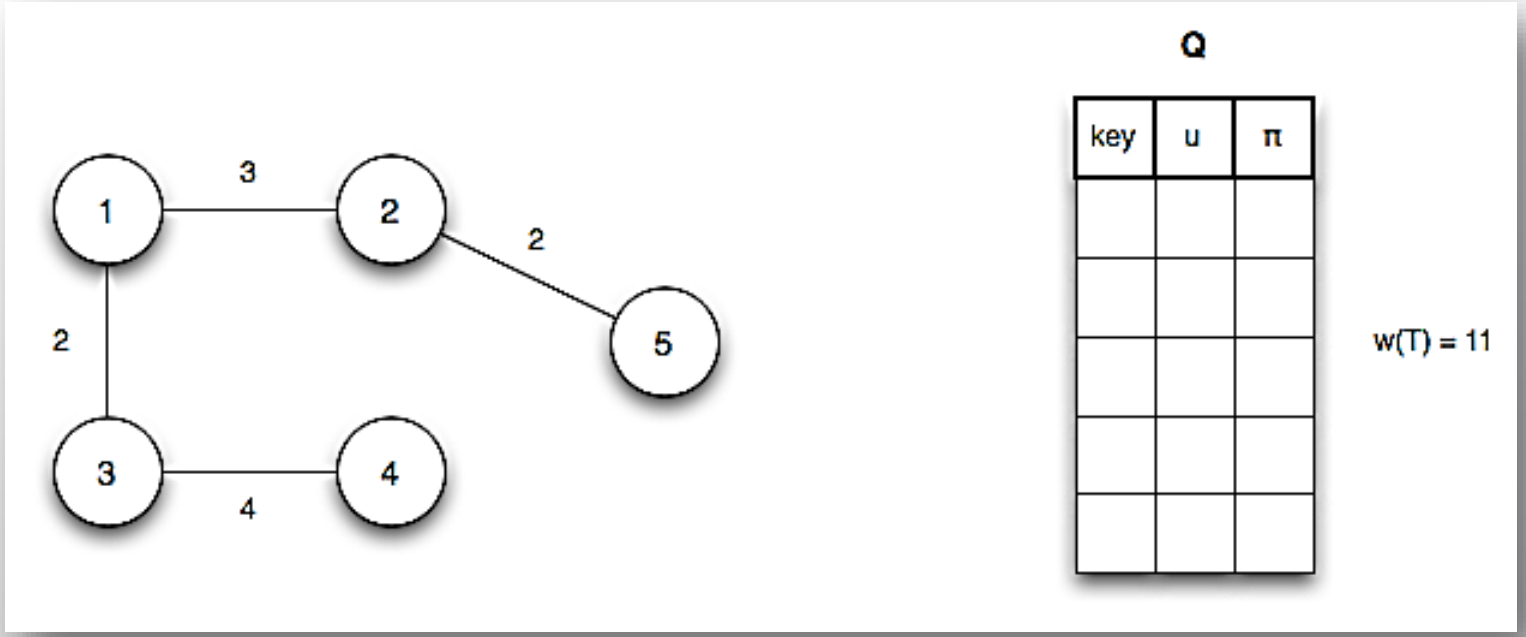
Adım 4

- Kuyruktan düğüm 5'i al, T'ye (u2,u5) kenarını ekle. Q'da güncelleme yok.



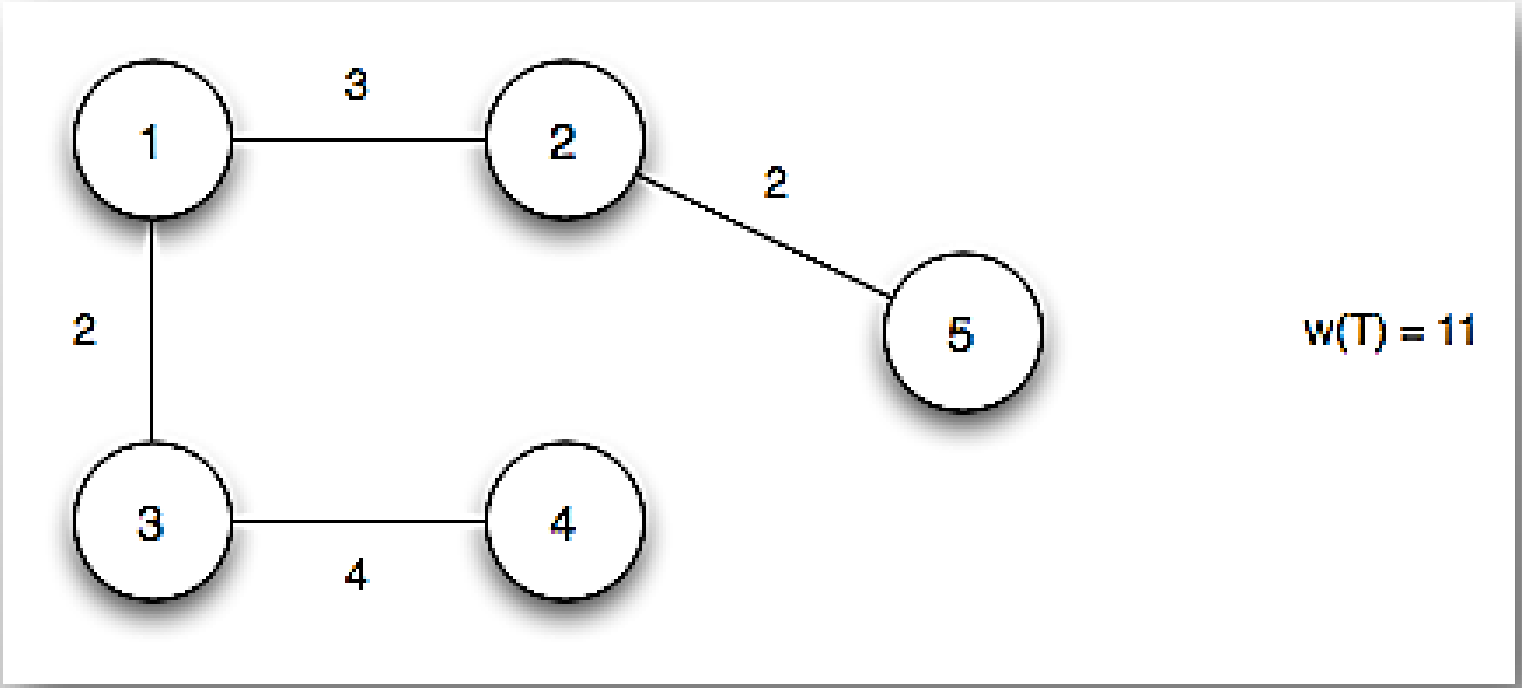
Adım 5

- Kuyruktan düğüm 4'ü al, T'ye (u3,u4) kenarını ekle. Q'da güncelleme yok.

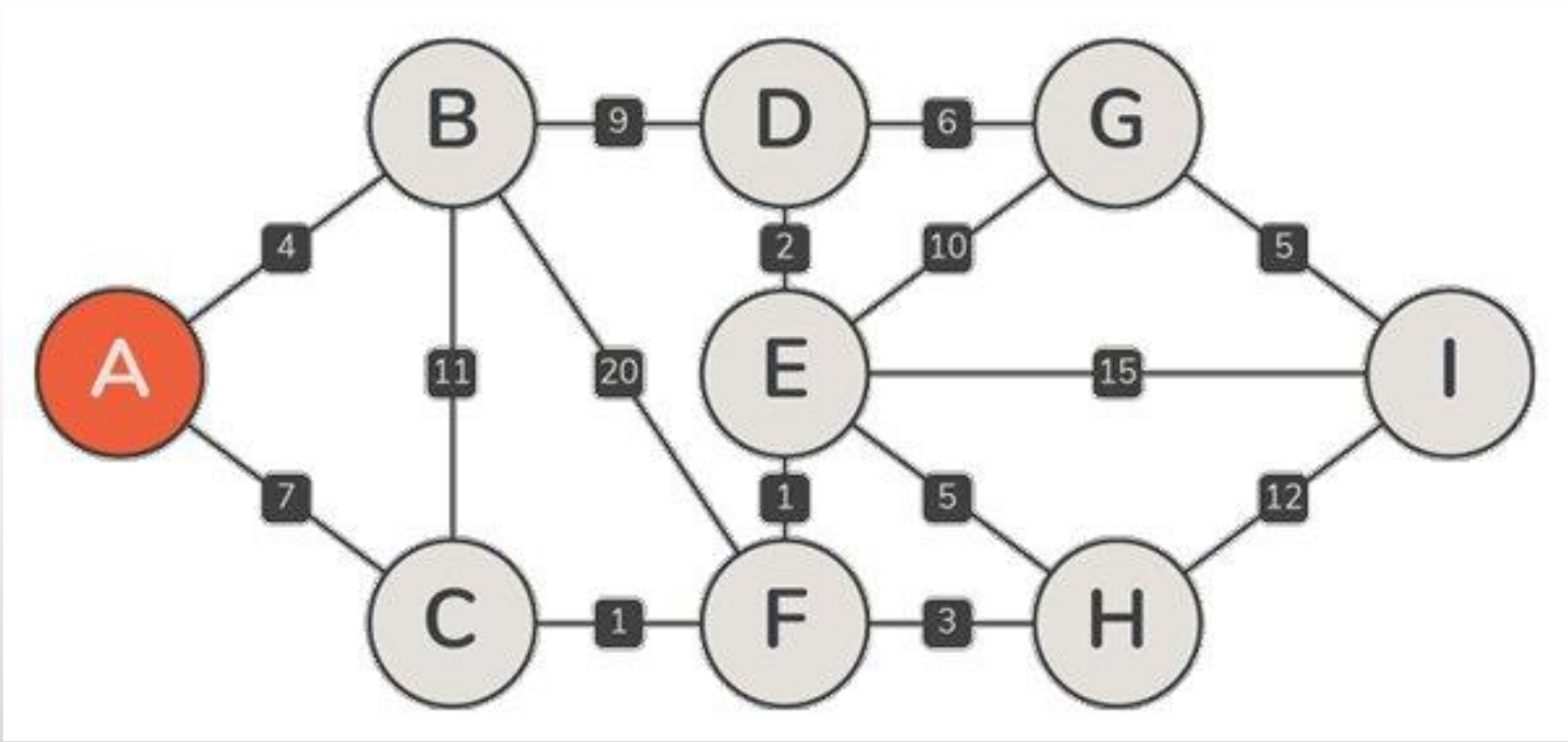


Son Durum

- Q = ∅



Örnek Prim



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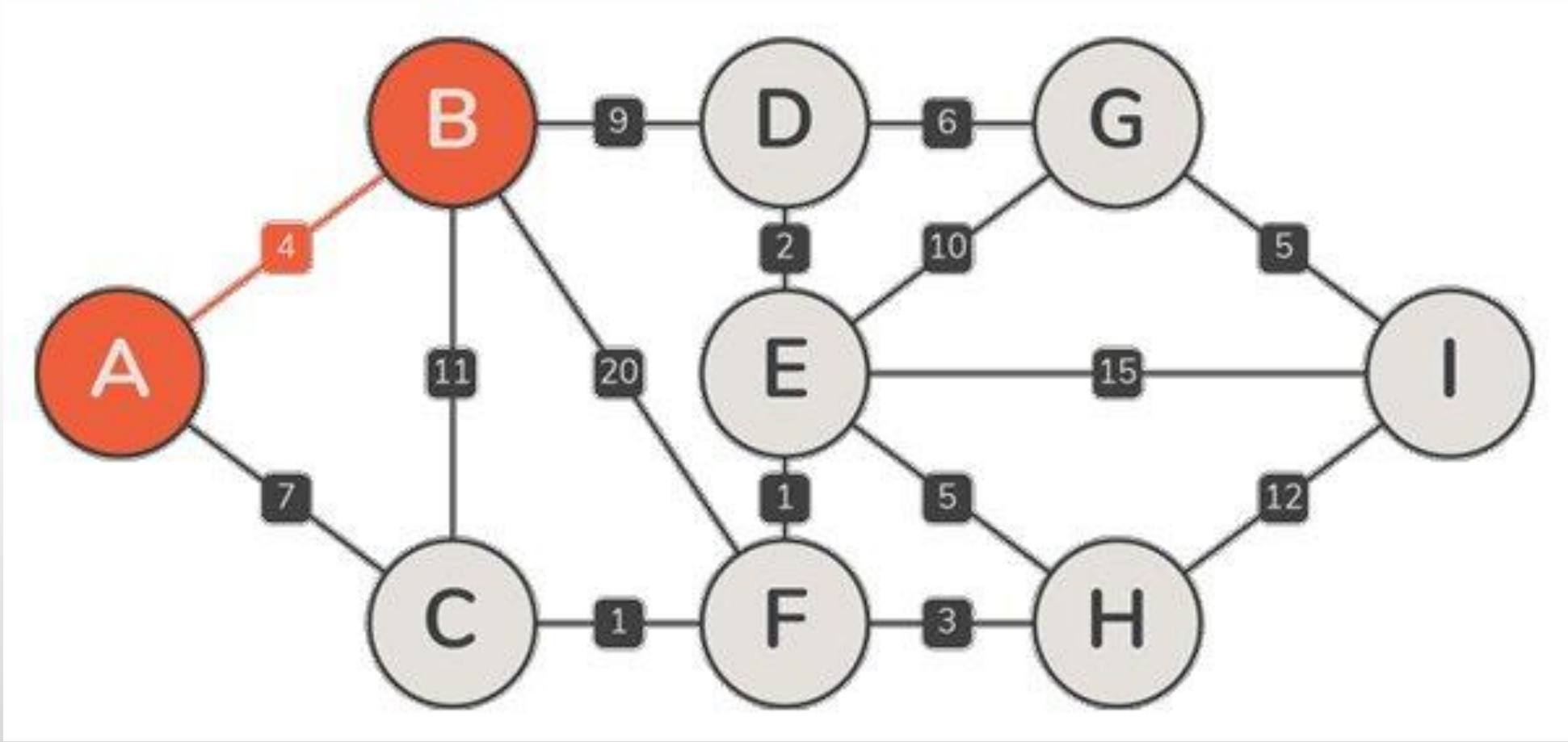

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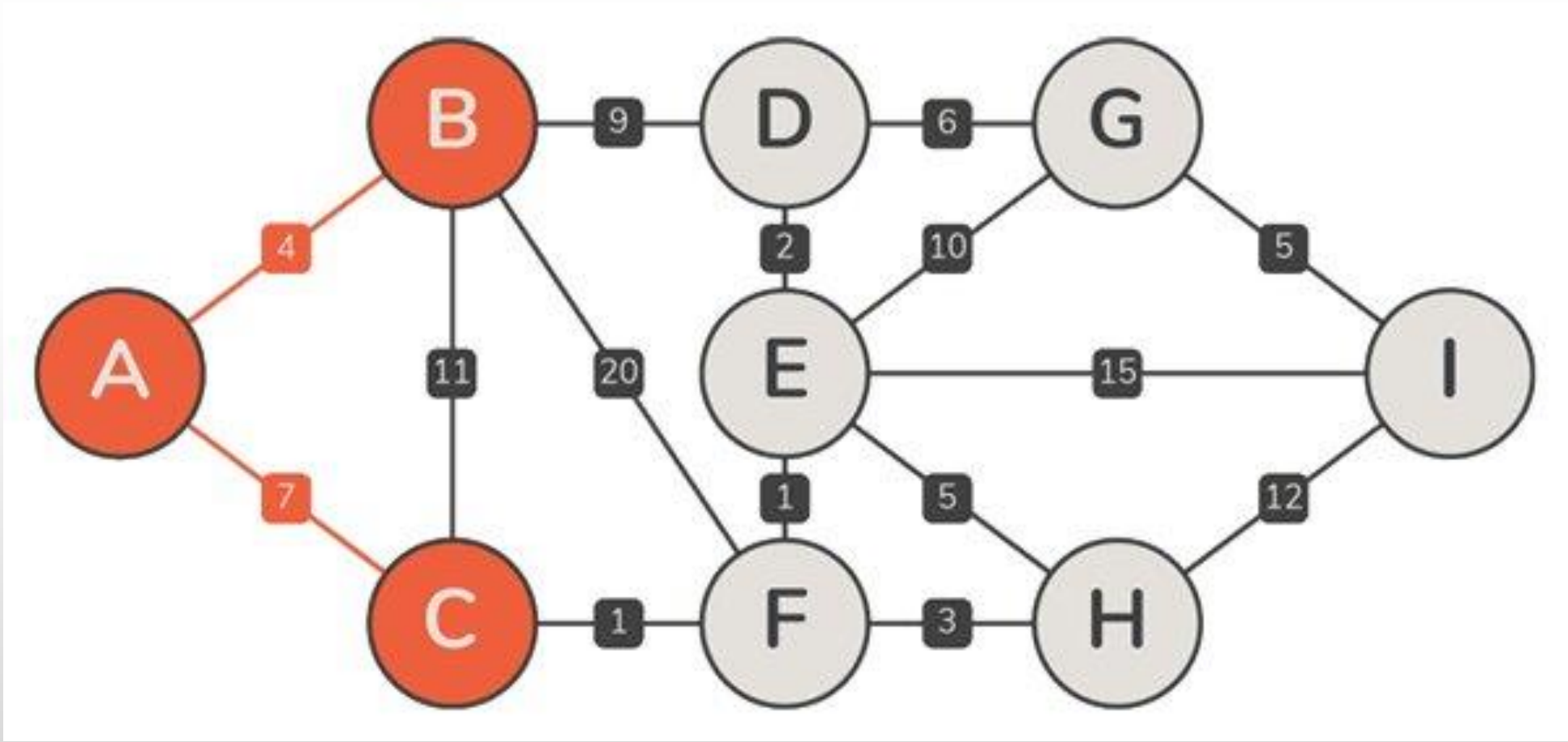

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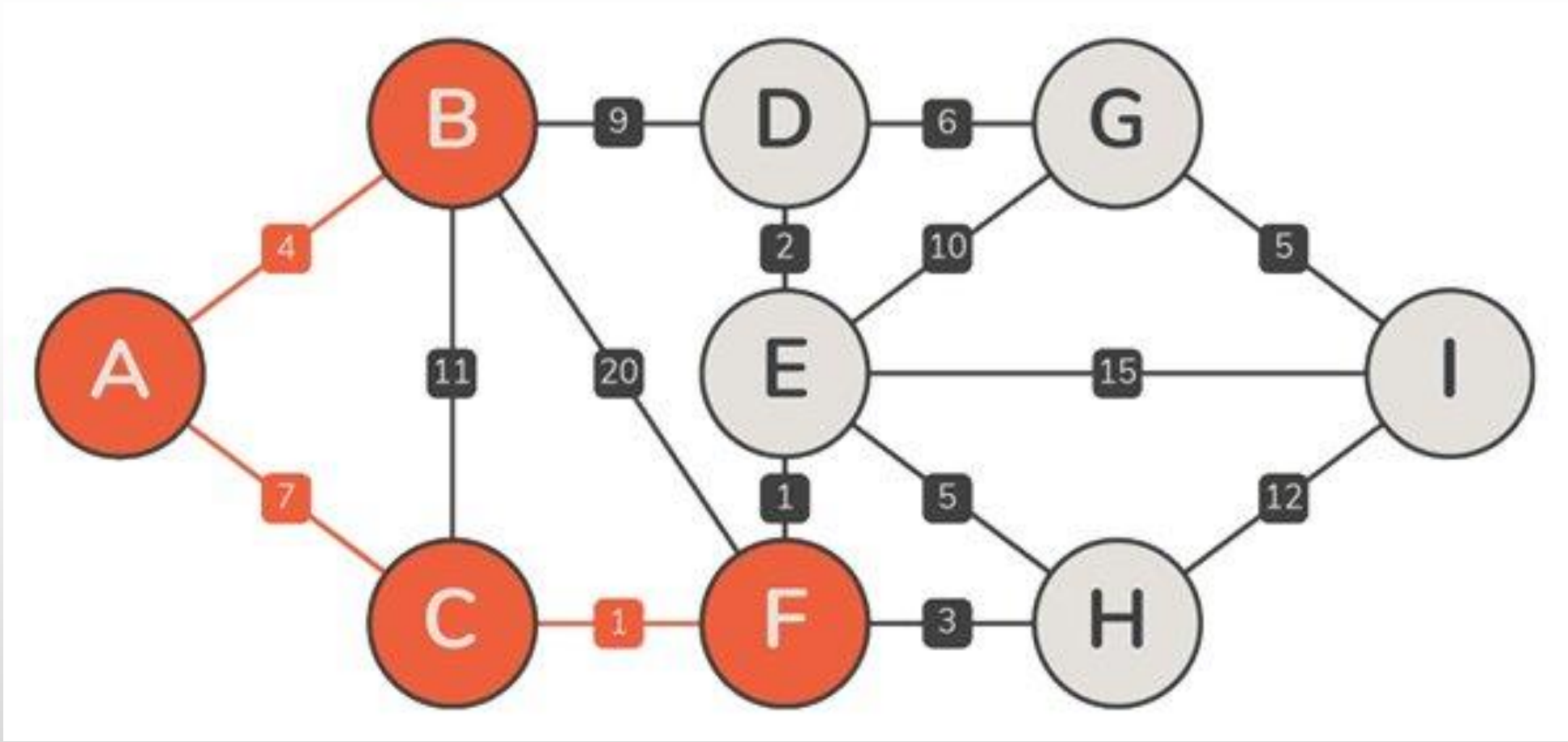

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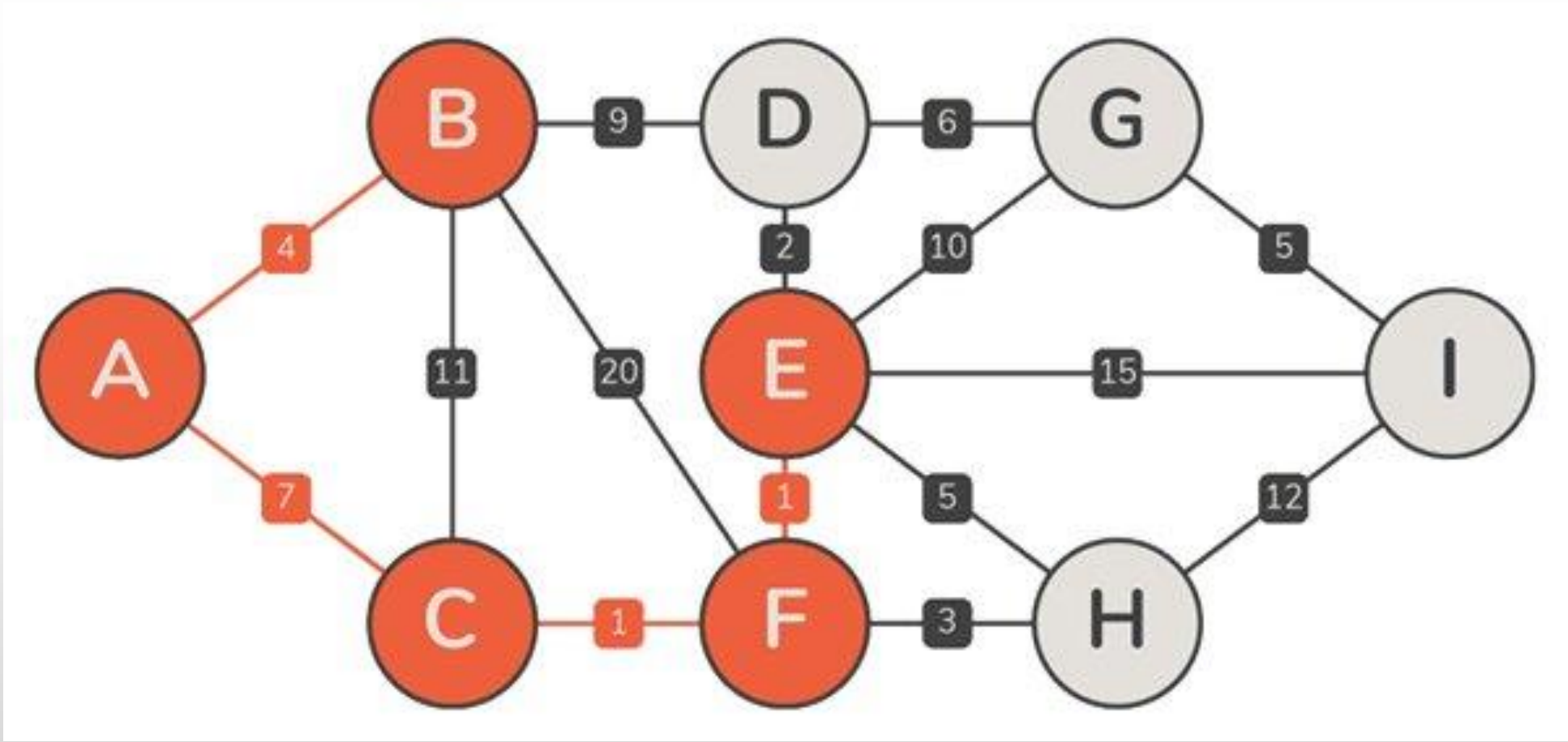

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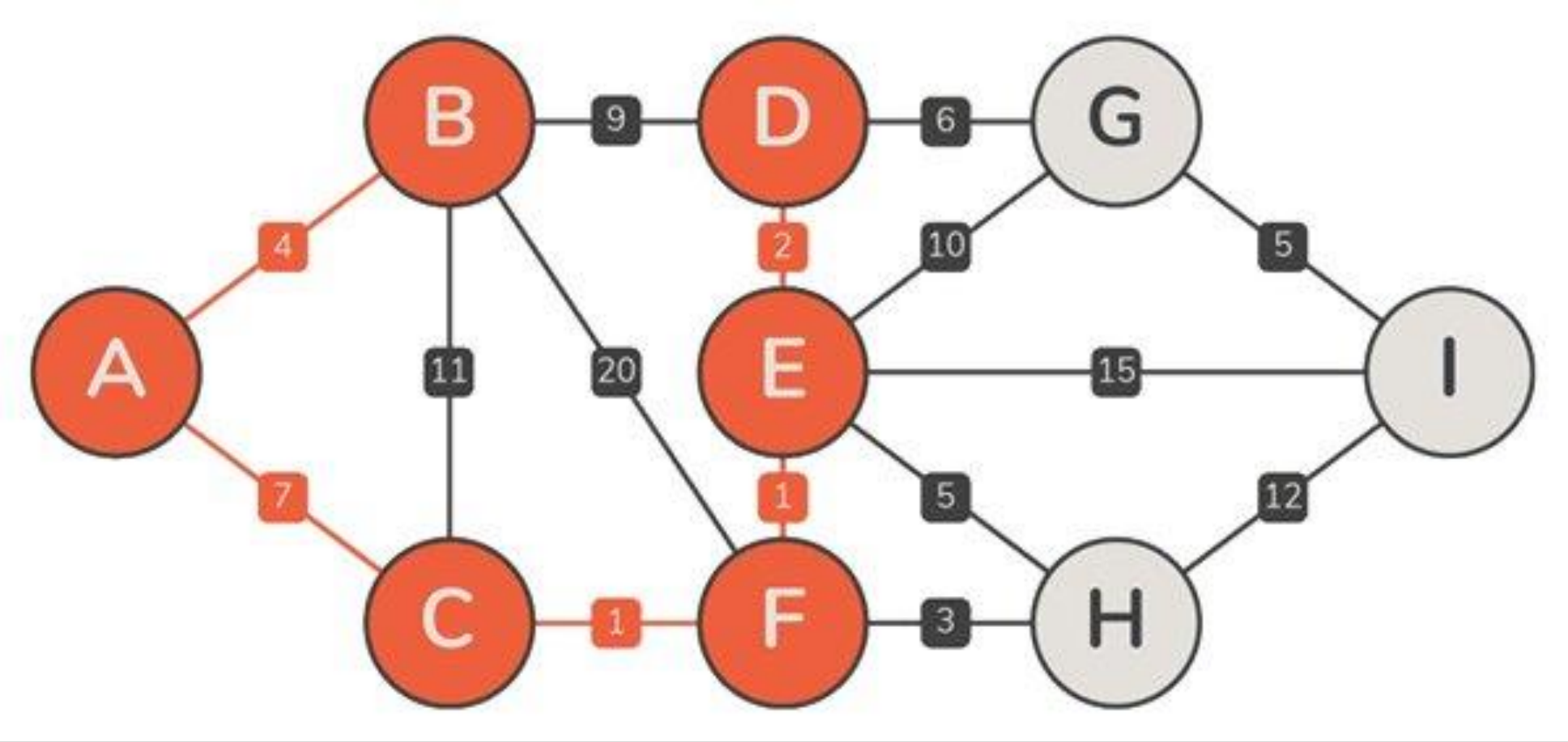

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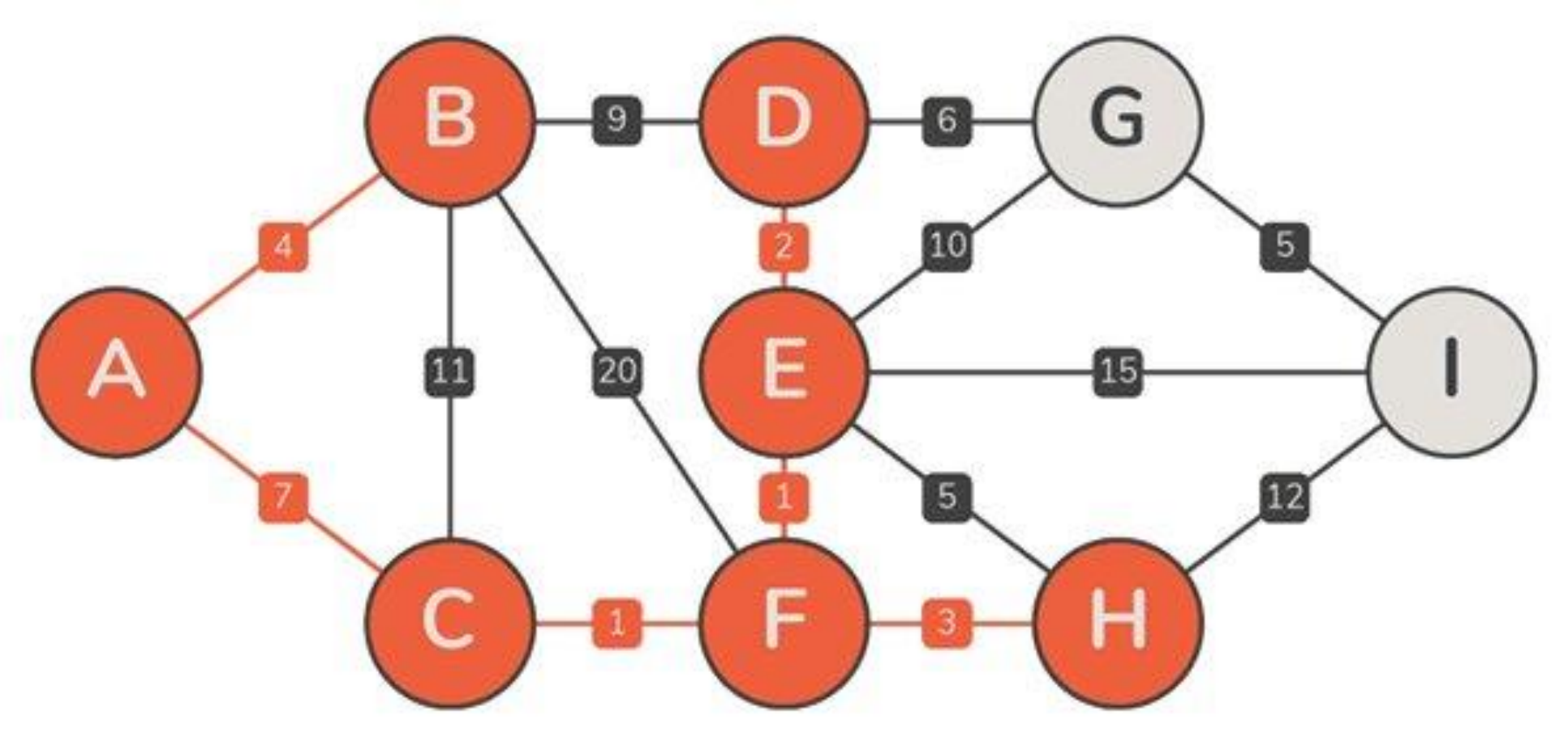

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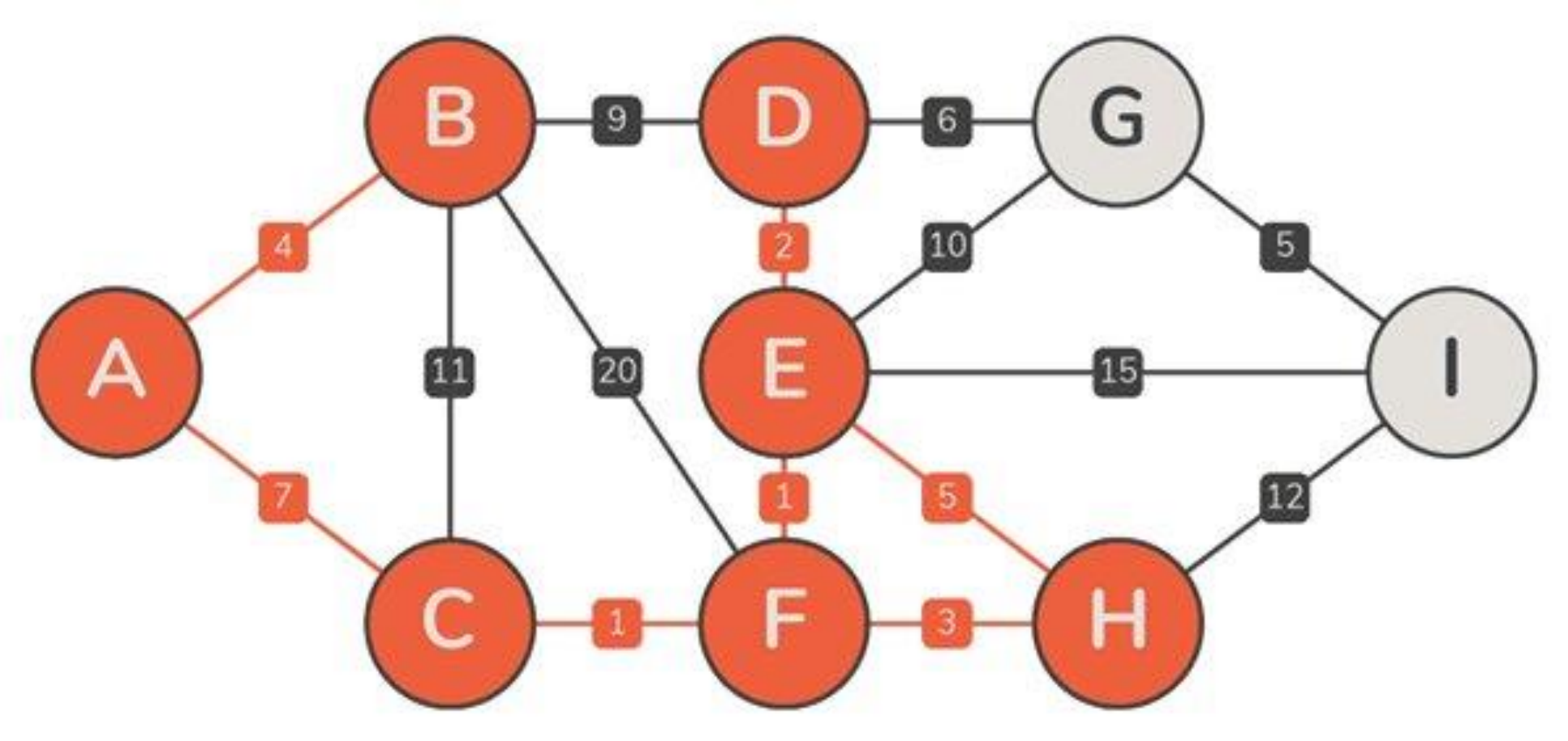

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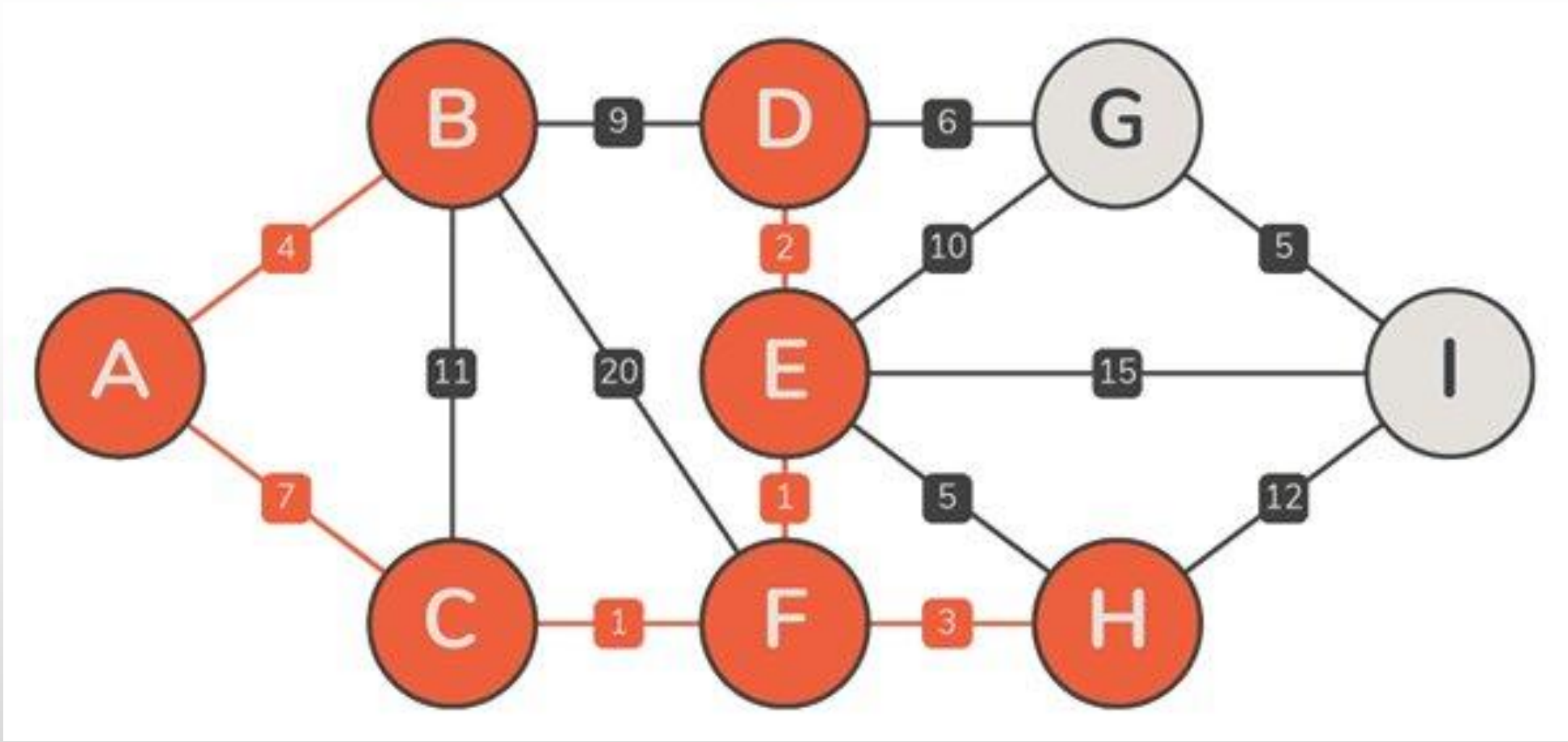

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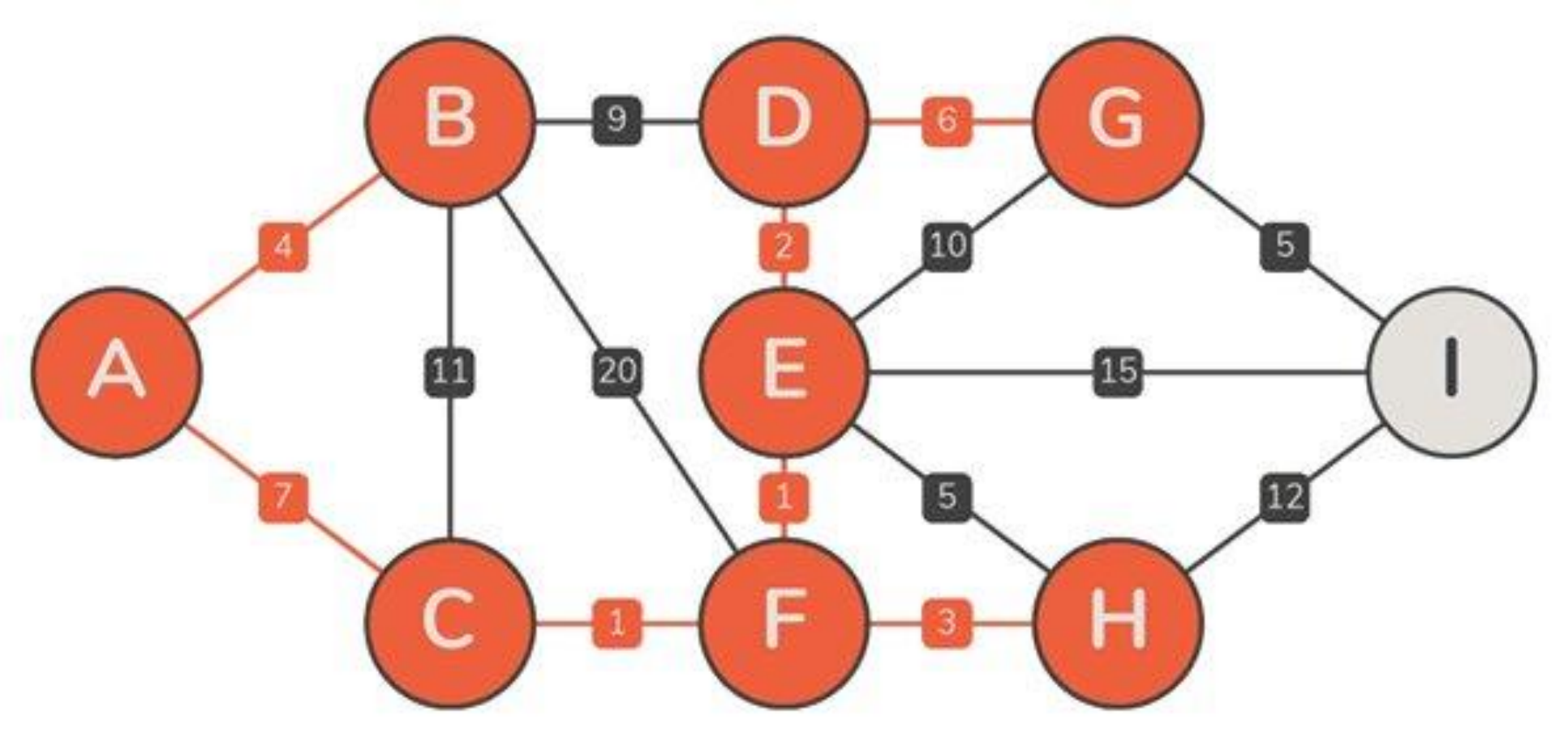

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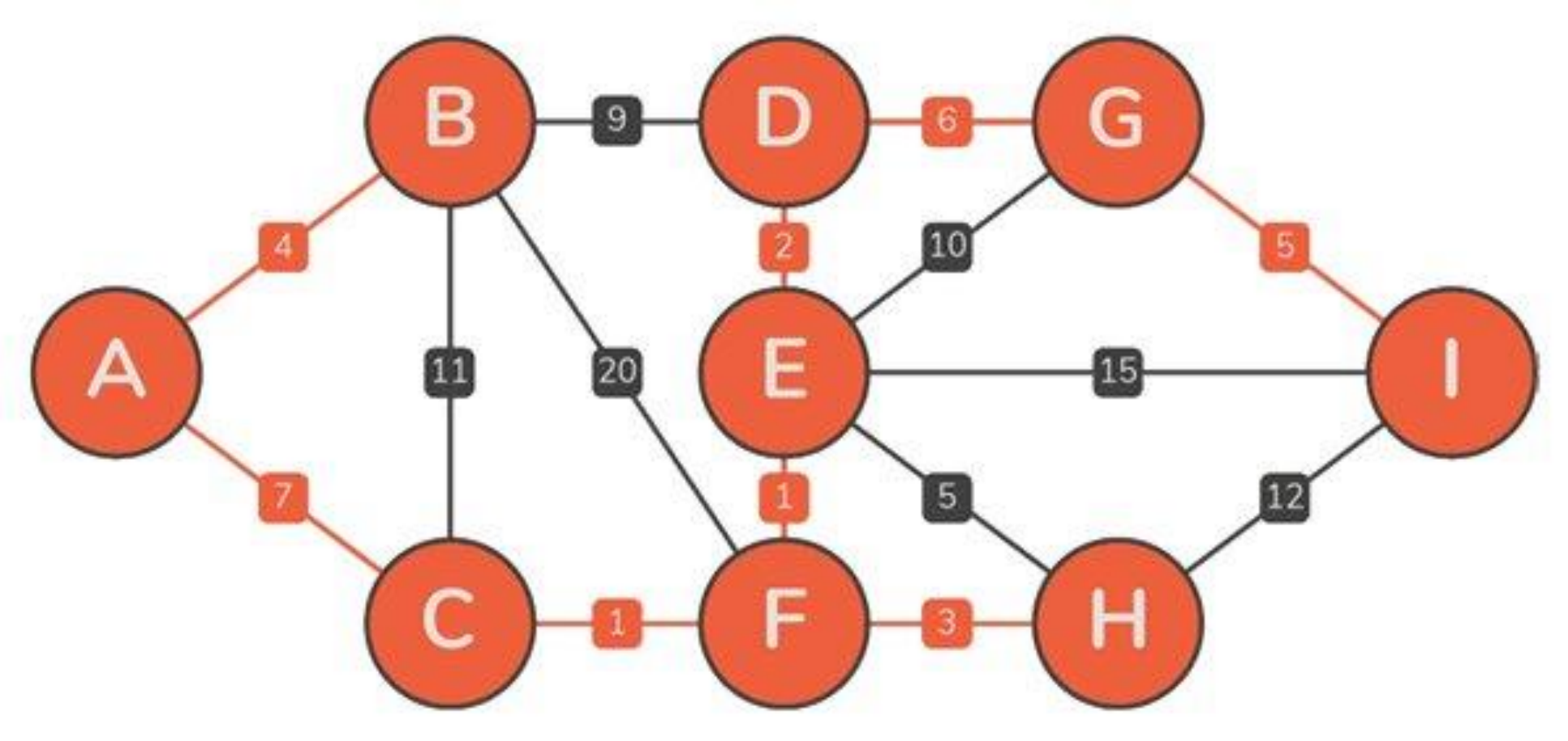


Boruvska Algoritması

- Minimum kapsayan ağaçları bulmak için kullanılır.
- Çizgenin tüm düğümlerini kapsayan ve ağırlığı minimum alt çizgeyi bulur.
- Nasıl Çalışır?
  - Başlangıçta her düğüm birbirinden bağımsız kabul edilir.
  - Her adımda,
    - en küçük ağırlıklı kenar seçilir ve
    - bu kenarın bağladığı iki düğüm birleştirilir.
  - Bu işlem, tüm düğümler birleşinceye kadar devam eder.

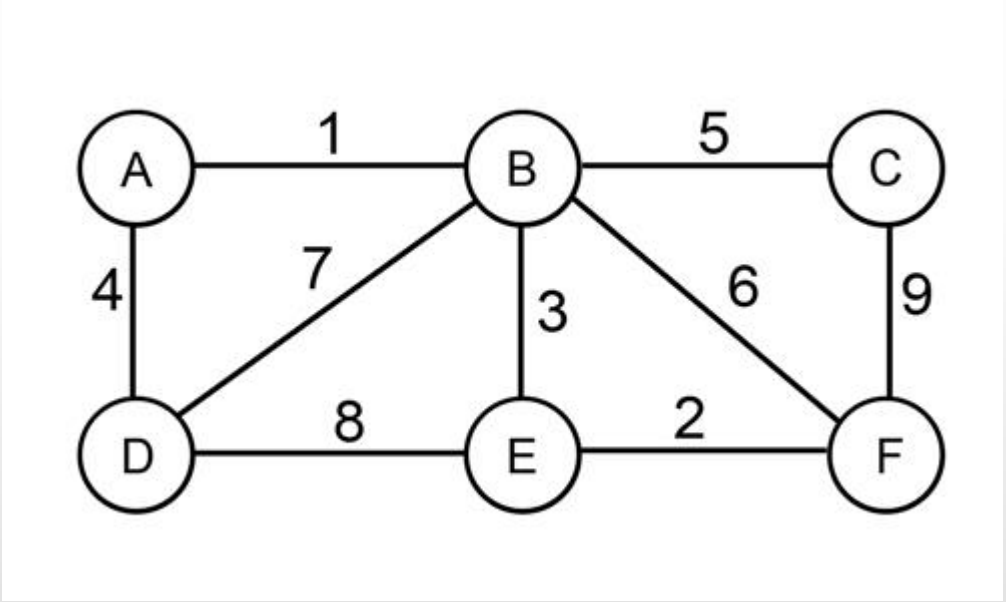
Algoritma Adımları

- Tüm kenarları ağırlıklarına göre sırala.
- Başlangıçta her düğümü tek başına bir küme olarak ele al.
- En küçük ağırlıklı kenarı seç ve bu kenarı bağlayan düğümleri birleştir.
- Her adımda oluşan alt küme ağacının ağırlığını güncelle.
- Tüm düğümler birleşinceye kadar 3 ve 4. adımları tekrarla.

Özellikleri

- Ağırlık sıralaması yaparak minimum ağaçları bulur.
- Greedy (Açgözlü) bir algoritmadır.
- Karmaşıklığı *O(E log V)*'dir,
  - E kenar sayısı,
  - V düğüm sayısıdır.

Örnek Boruvka



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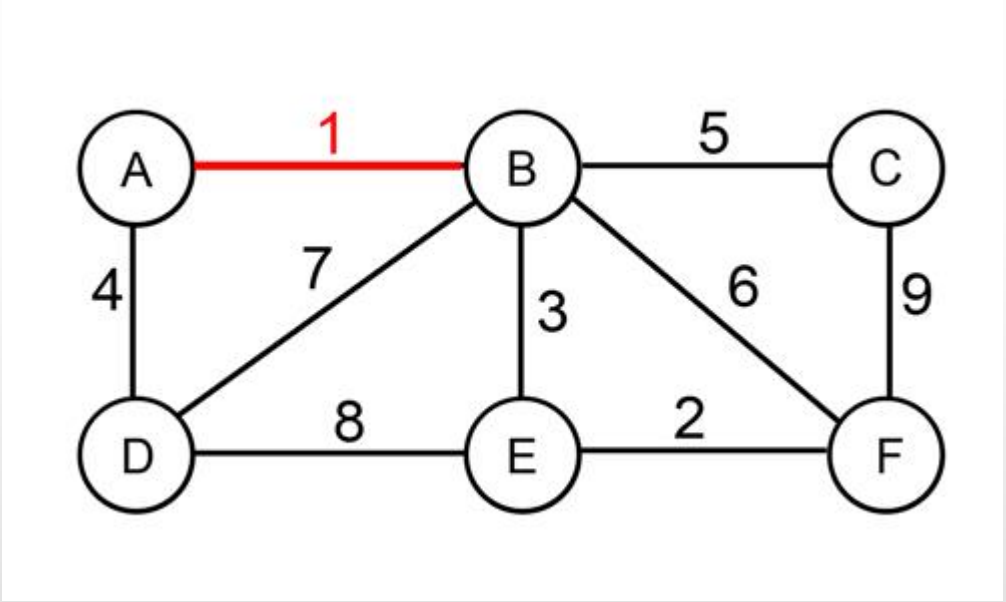
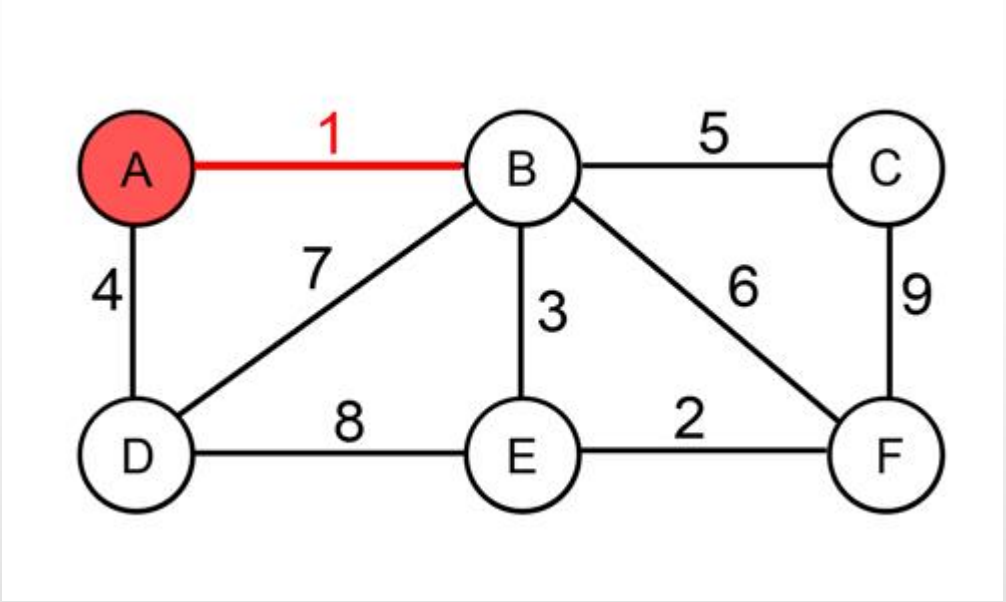


Örnek Boruvka



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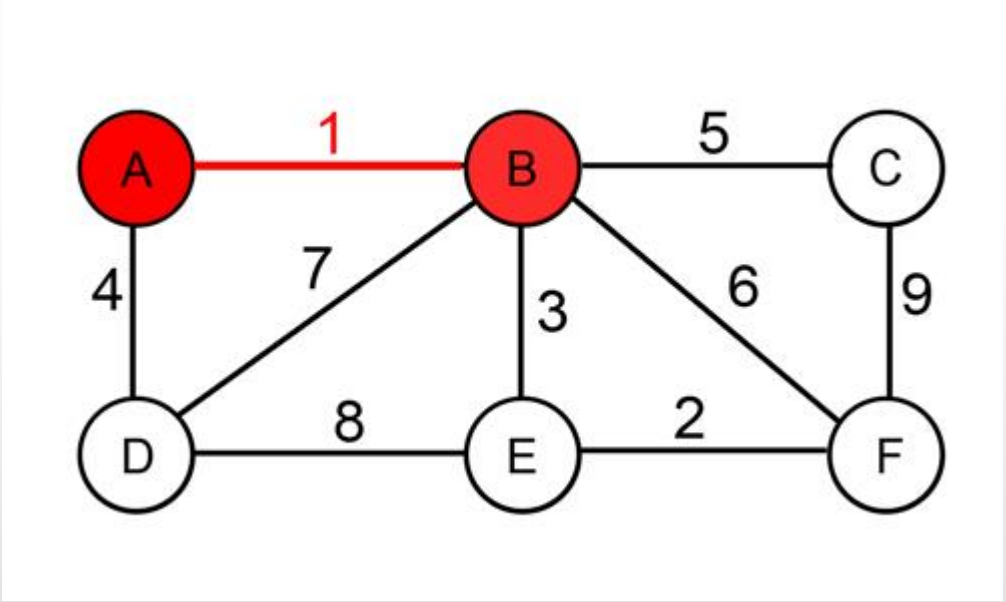

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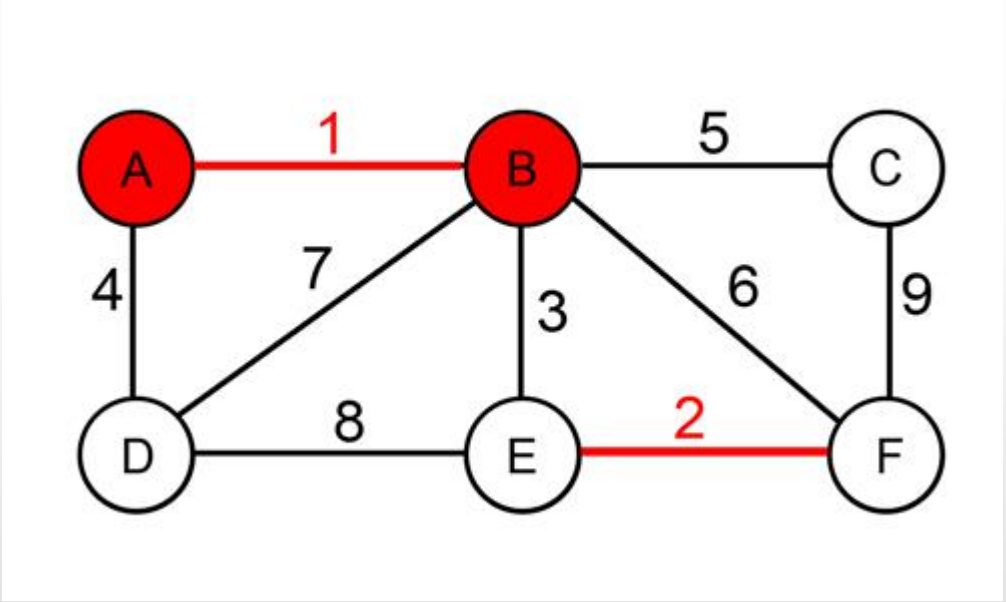

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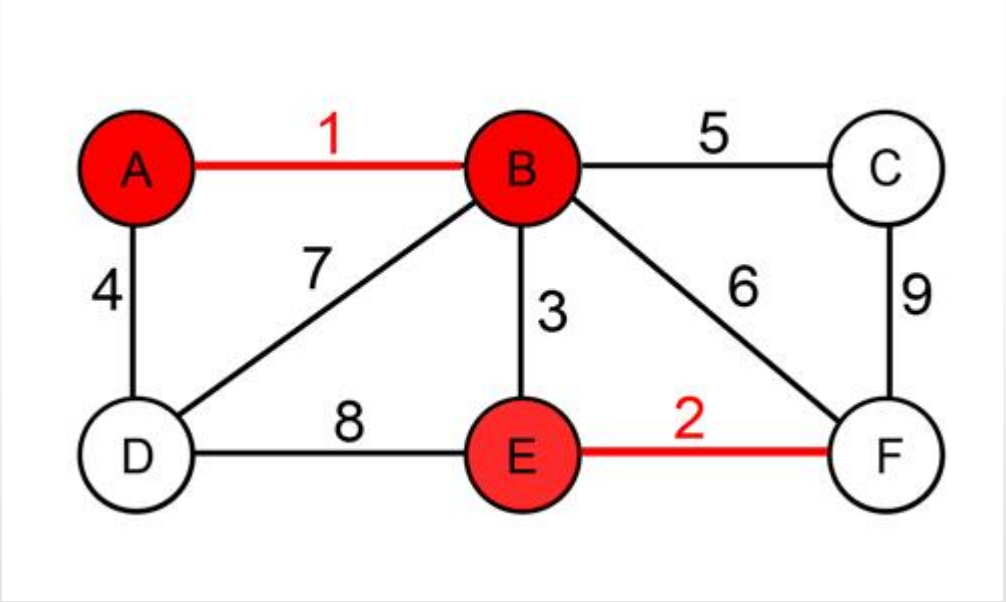

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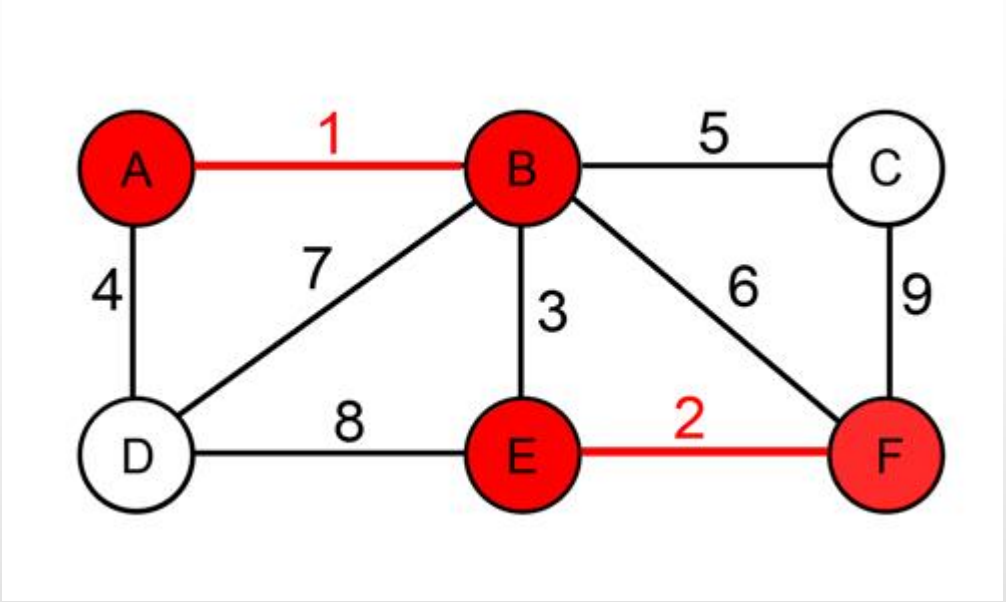

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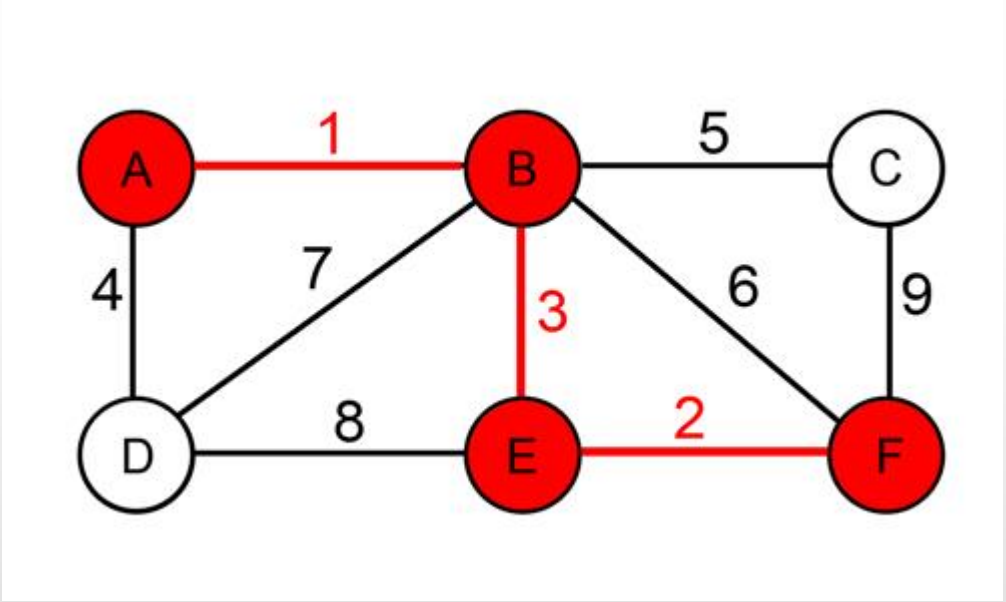

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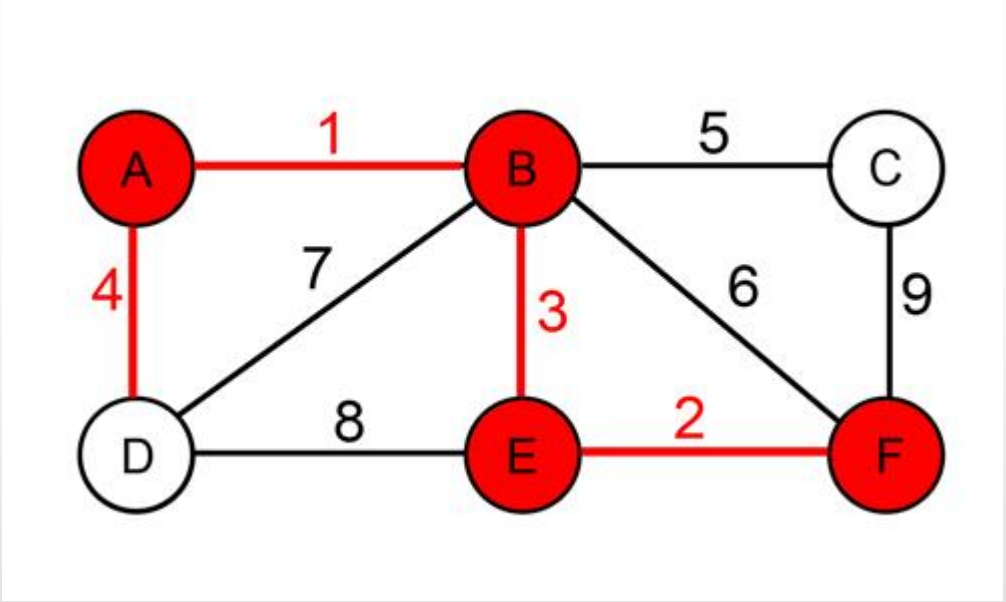

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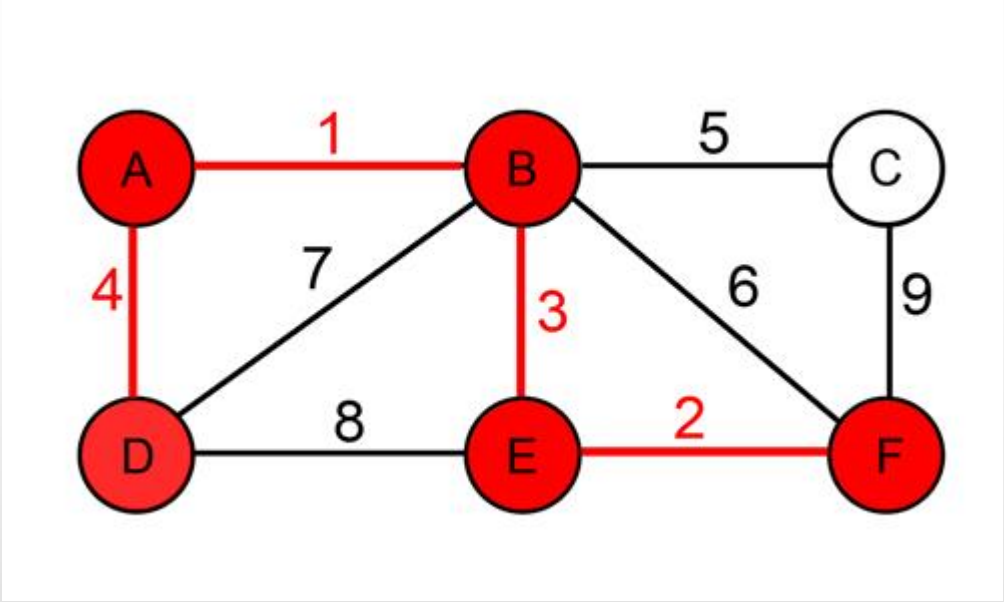

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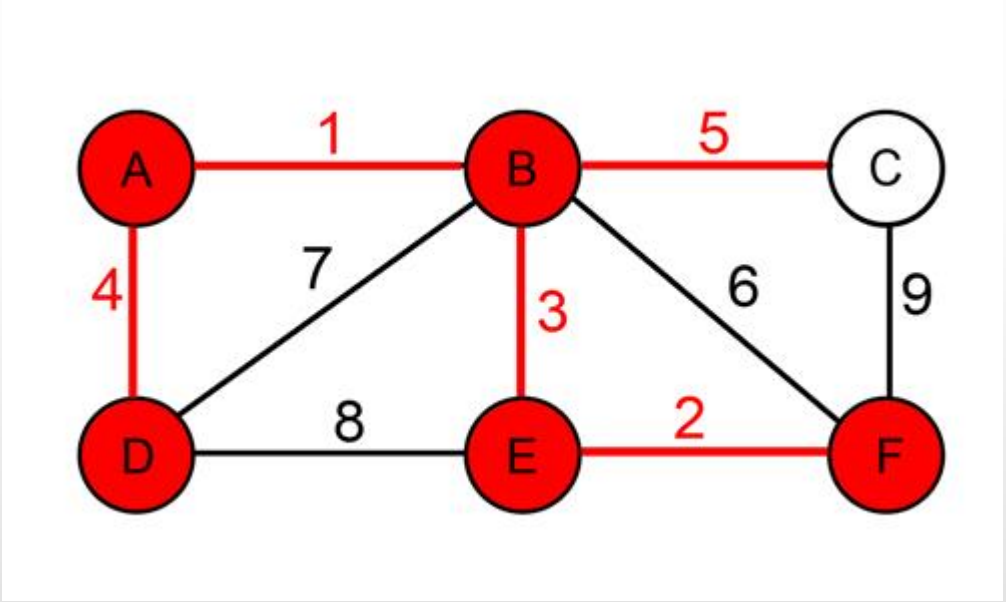

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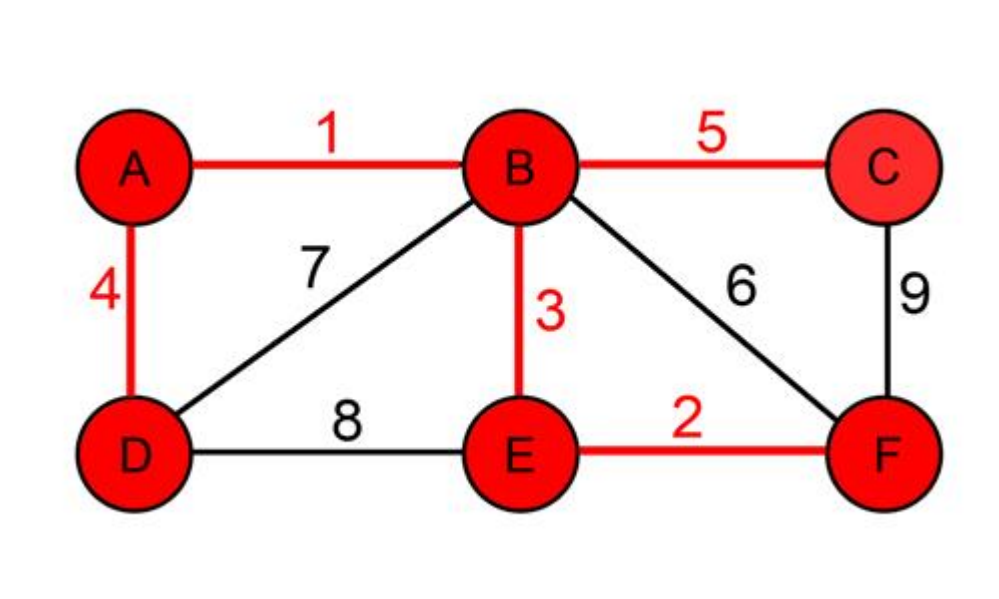

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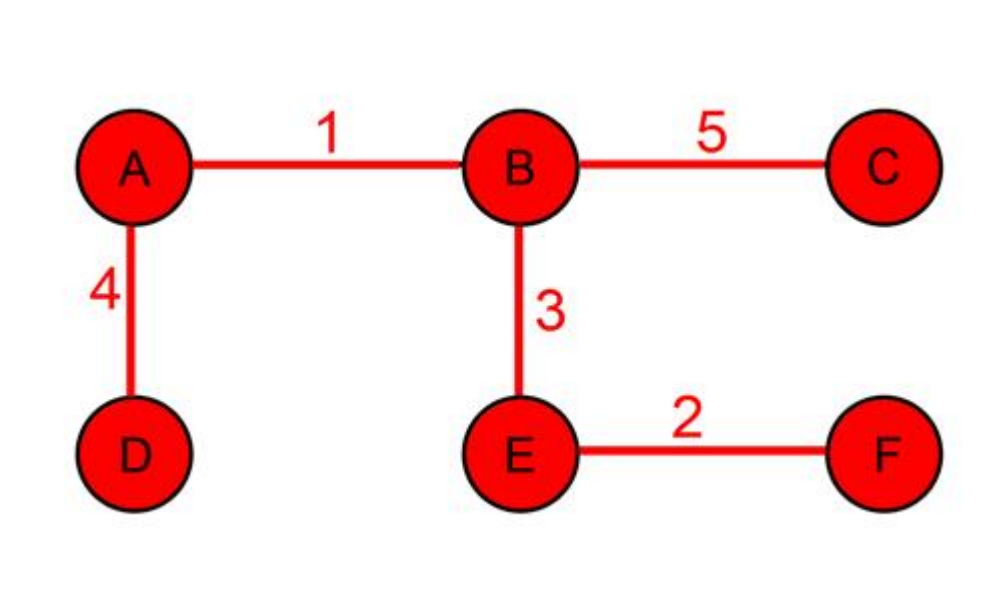


Sollin Algoritması

- Başlangıçta her düğüm birbirinden bağımsız olarak kabul edilir.
- Her adımda,
  - her düğüm için en düşük ağırlıklı kenar seçilir ve
  - bu kenarın bağladığı düğümler birleştirilir.
- Bu işlem, tüm düğümler birleşinceye kadar devam eder.

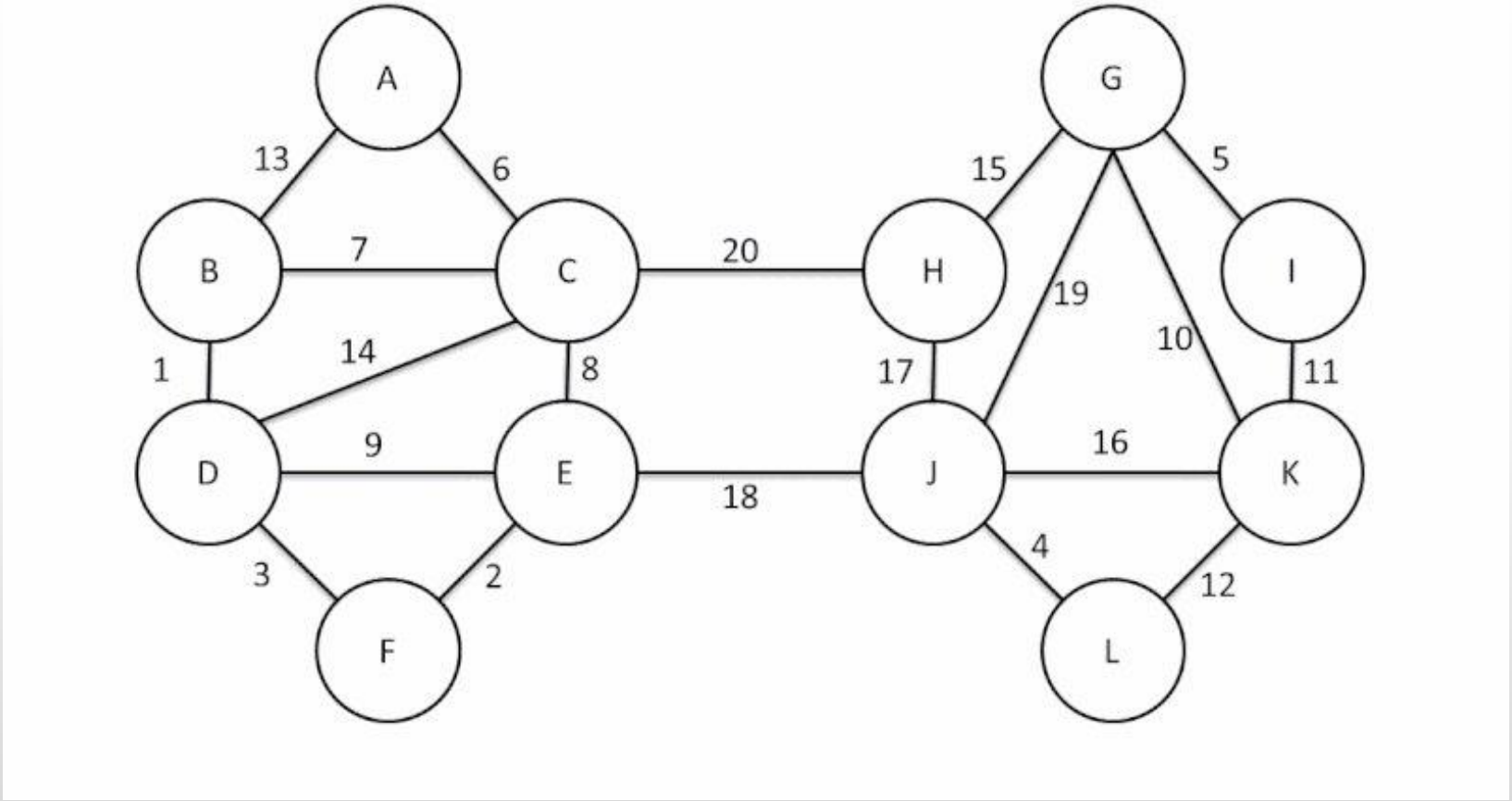
Algoritma Adımları

- Başlangıçta her düğümü tek başına bir küme olarak ele al.
- Her düğüm için, komşu kenarlar arasından en düşük ağırlıklı olanı seç.
- Seçilen kenarların her iki ucundaki düğümleri birleştir.
- Her adımda oluşan alt küme ağacının ağırlığını güncelle.
- Tüm düğümler birleşinceye kadar 2, 3 ve 4. adımları tekrarla.

Özellikleri

- Paralel işlem gücünden yararlanabilir.
- Diğer algoritmalara kıyasla daha hızlı çalışabilir.
- Zaman karmaşıklığı *O(E + V log V)*'dir,
  - E kenar sayısı,
  - V düğüm sayısıdır.

Boruvka's Algorithm



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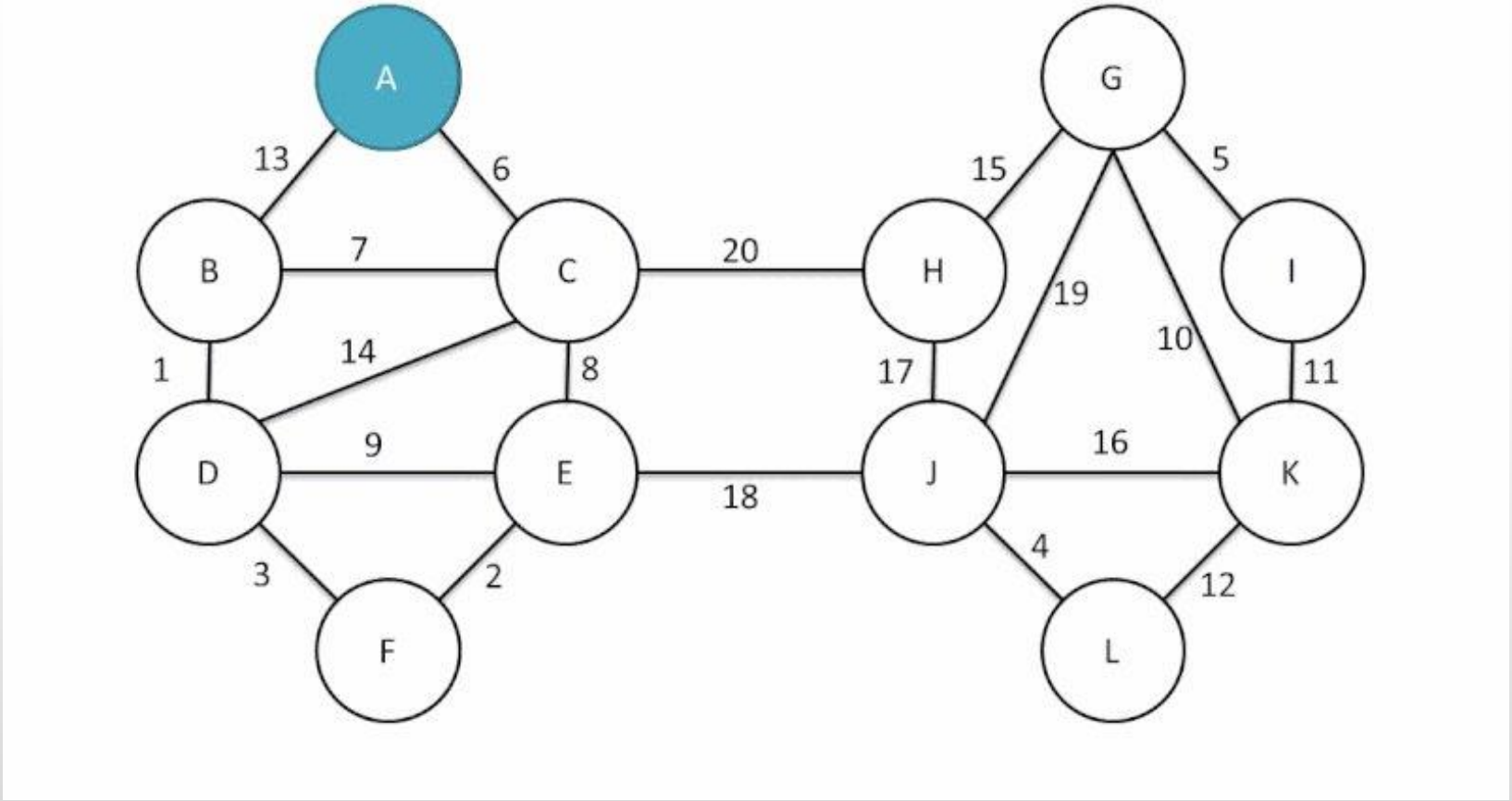

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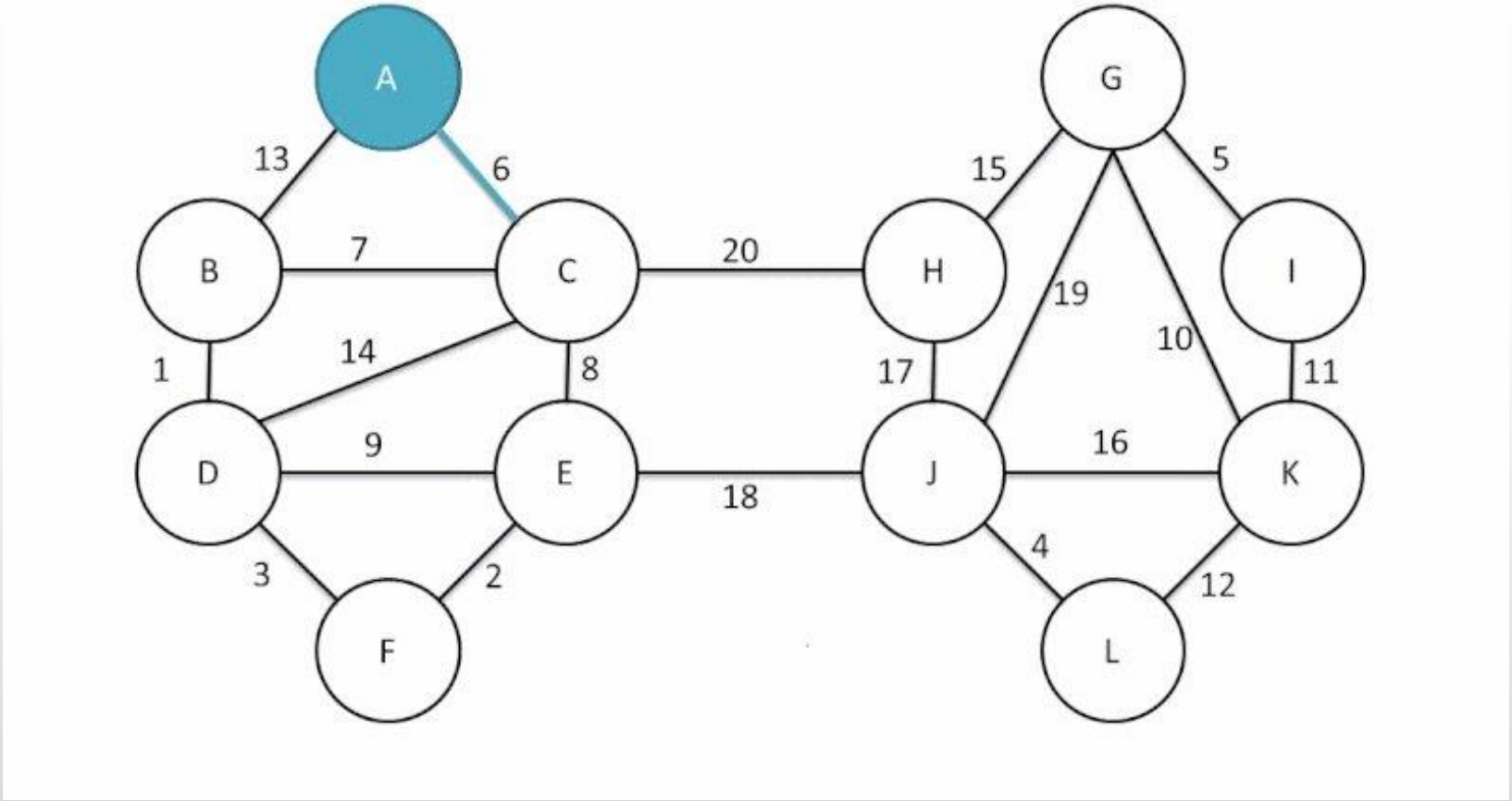

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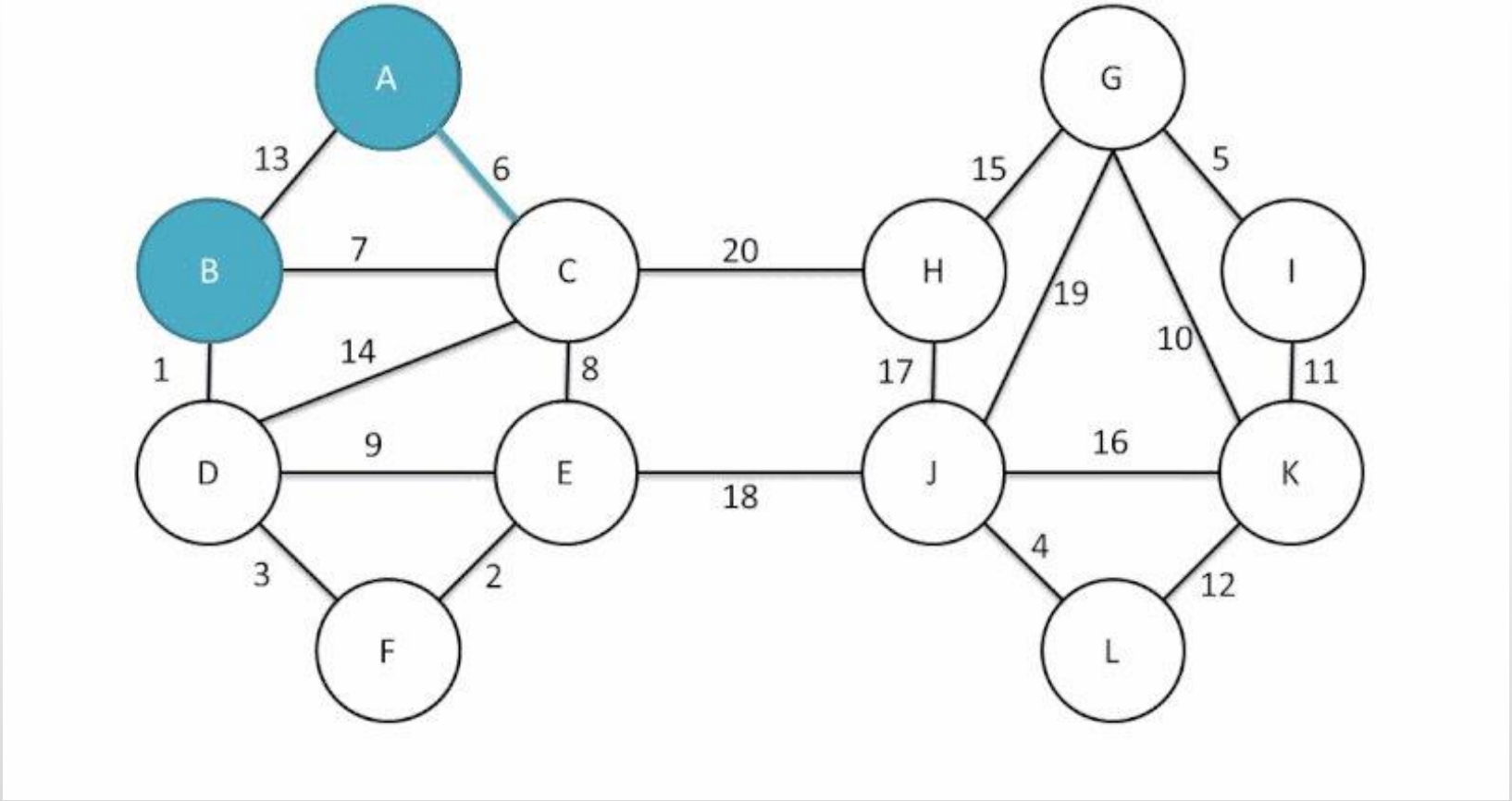

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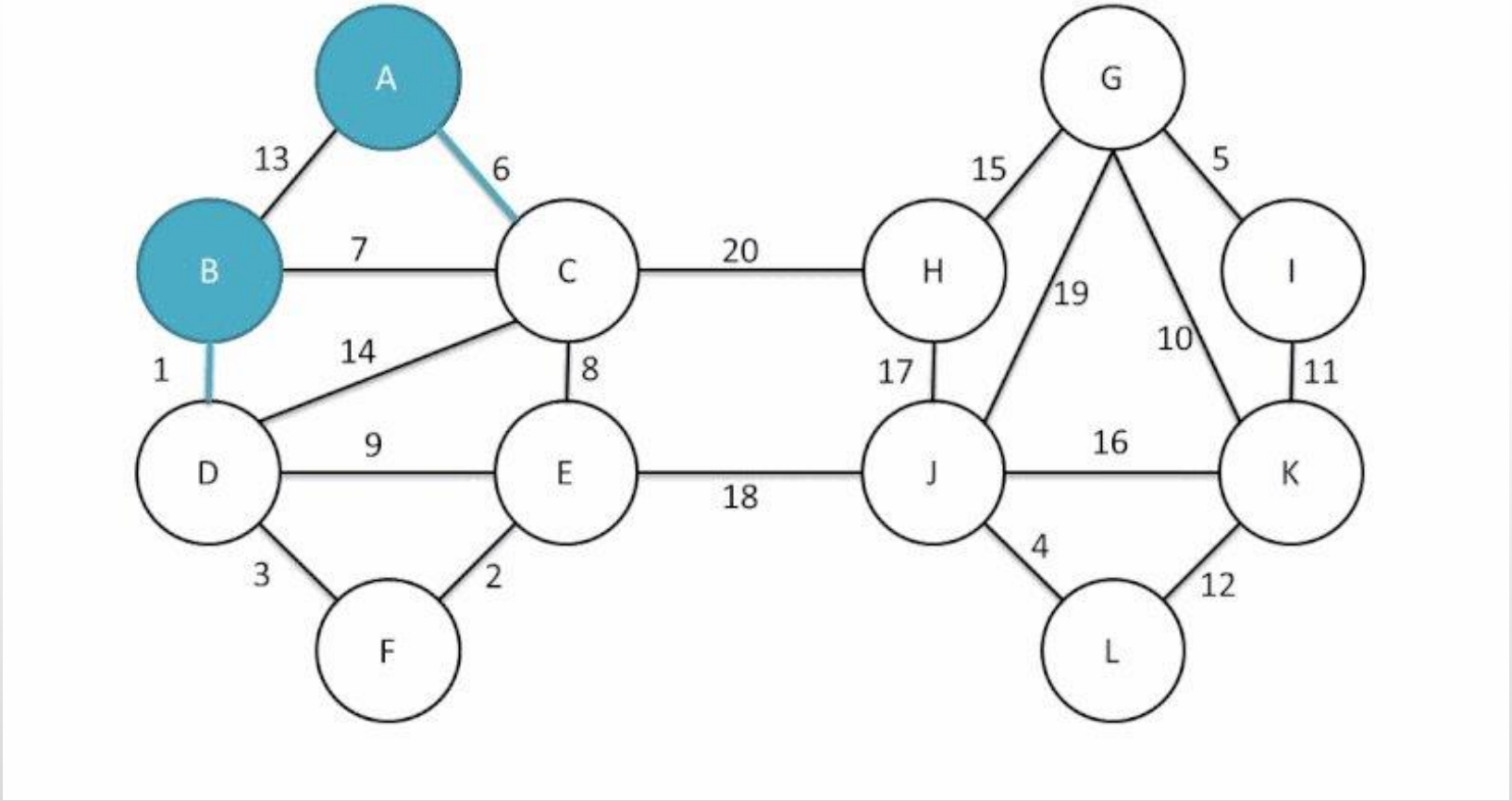

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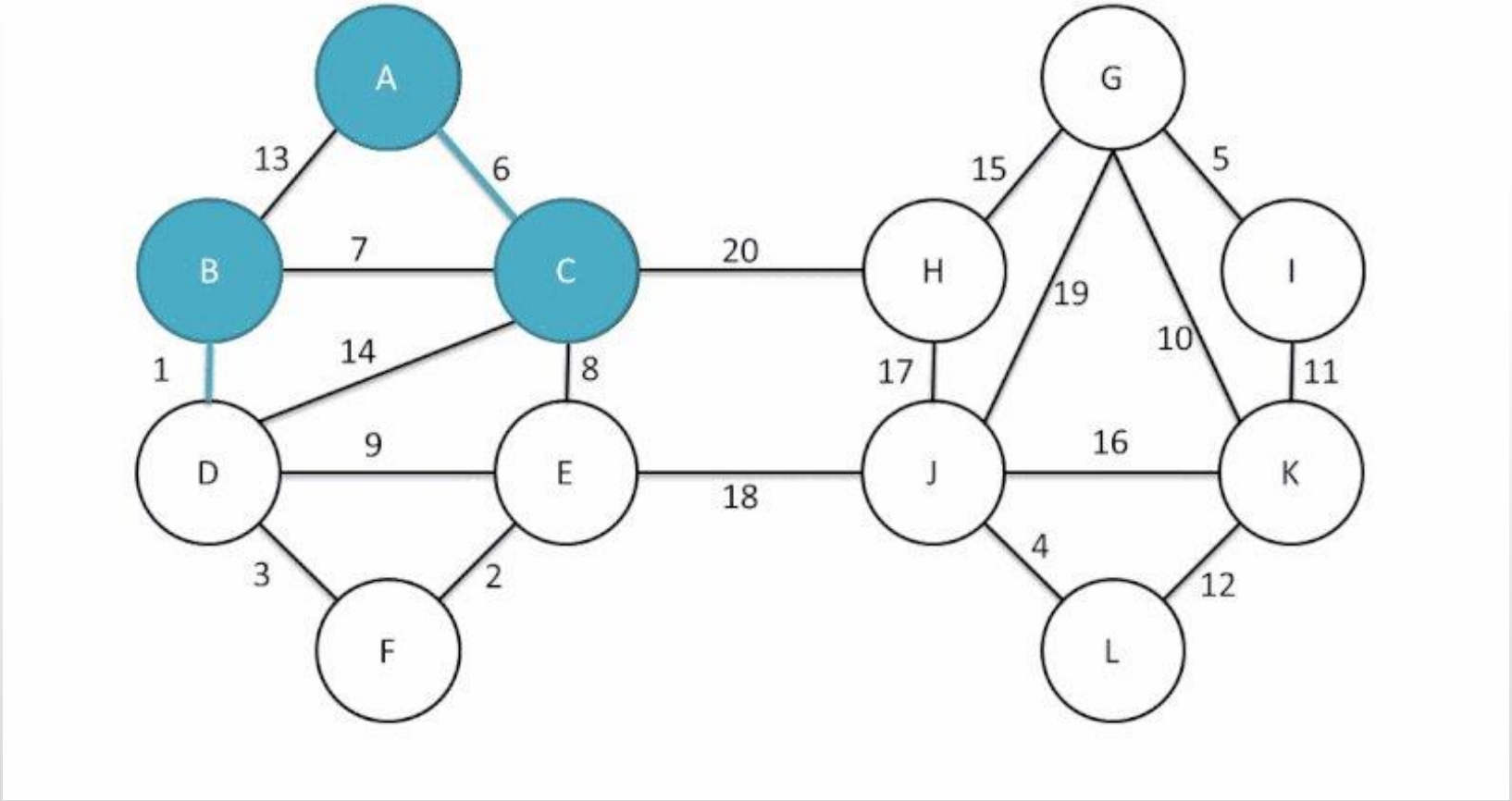

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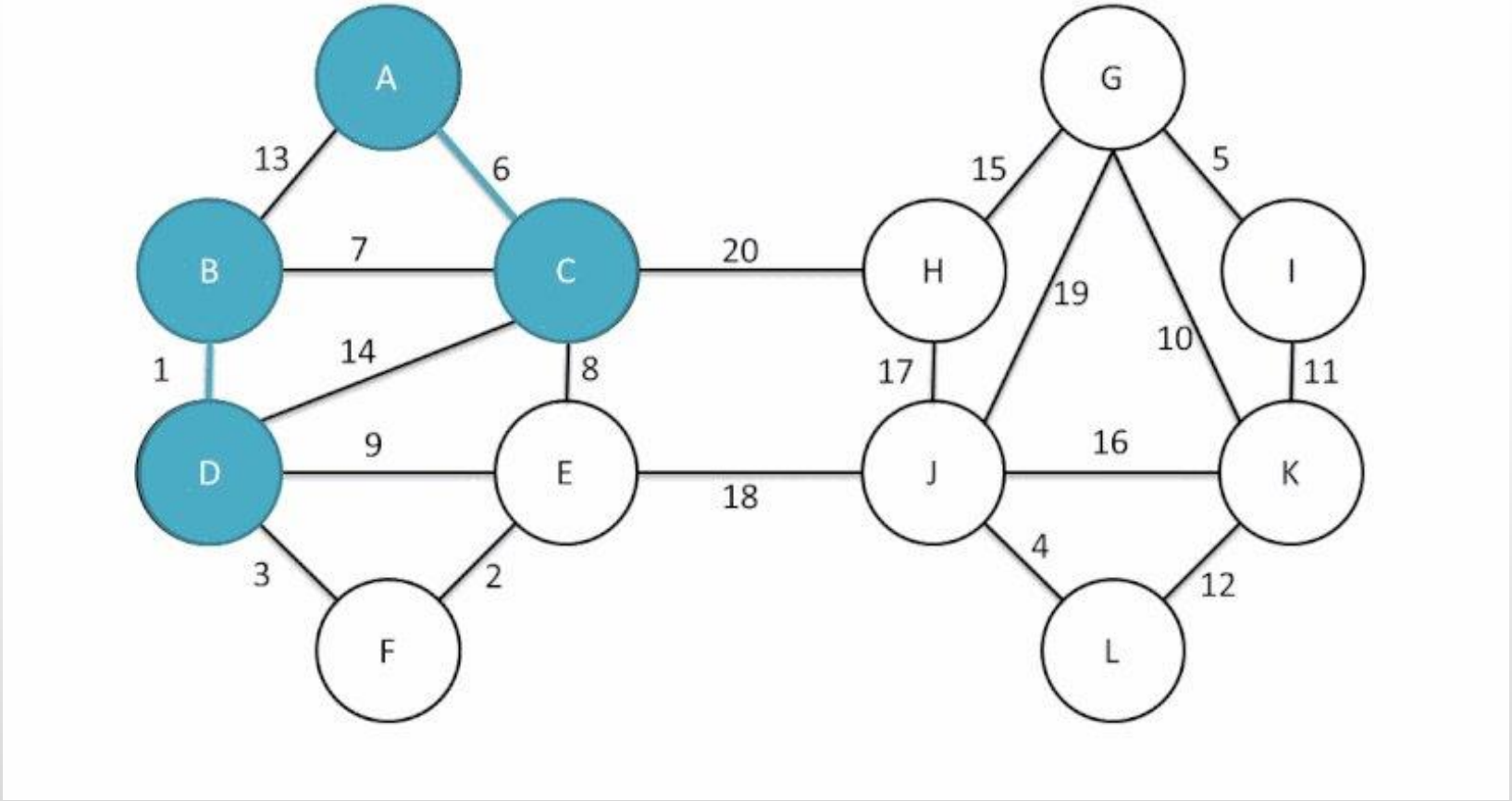

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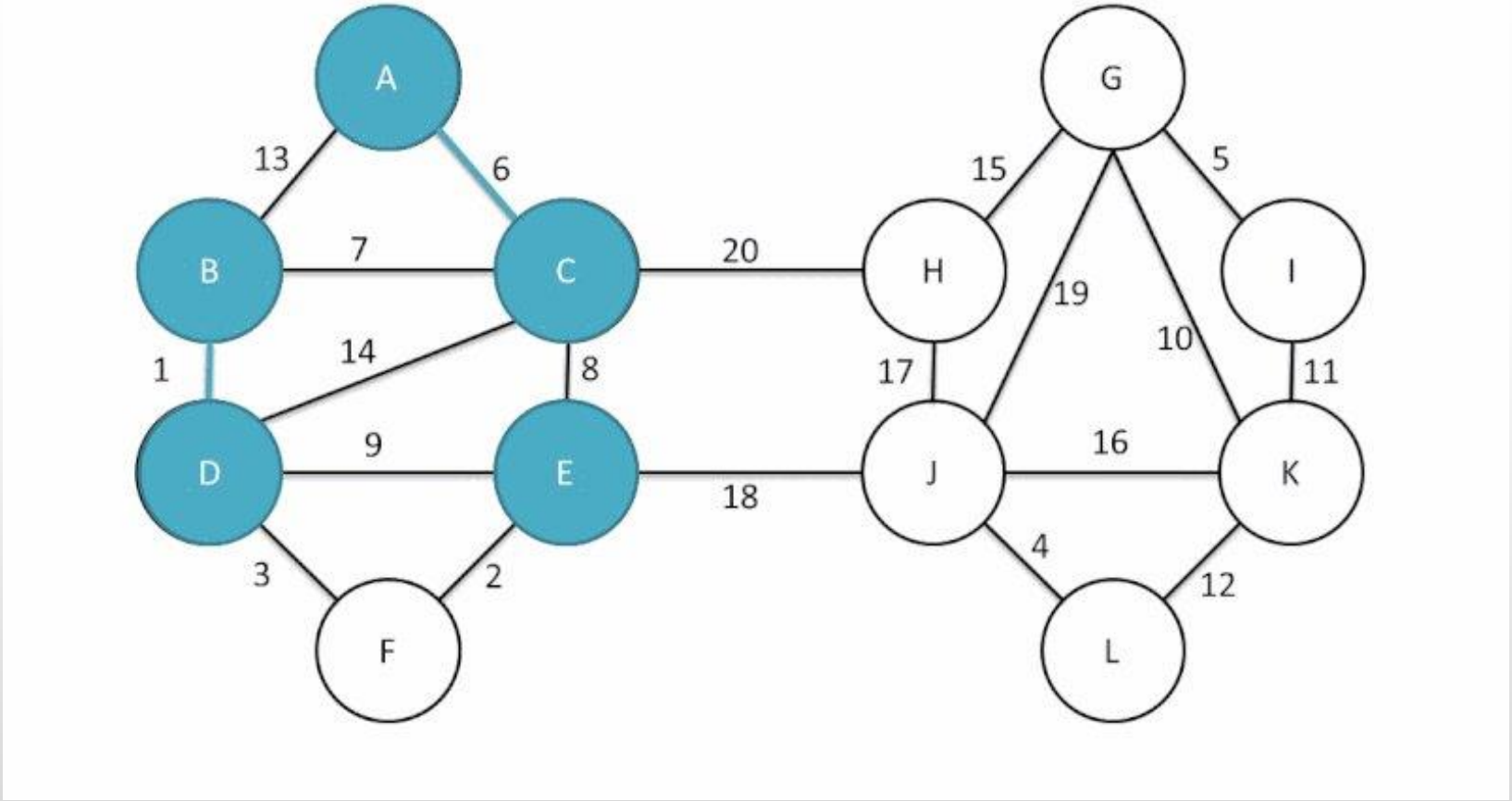

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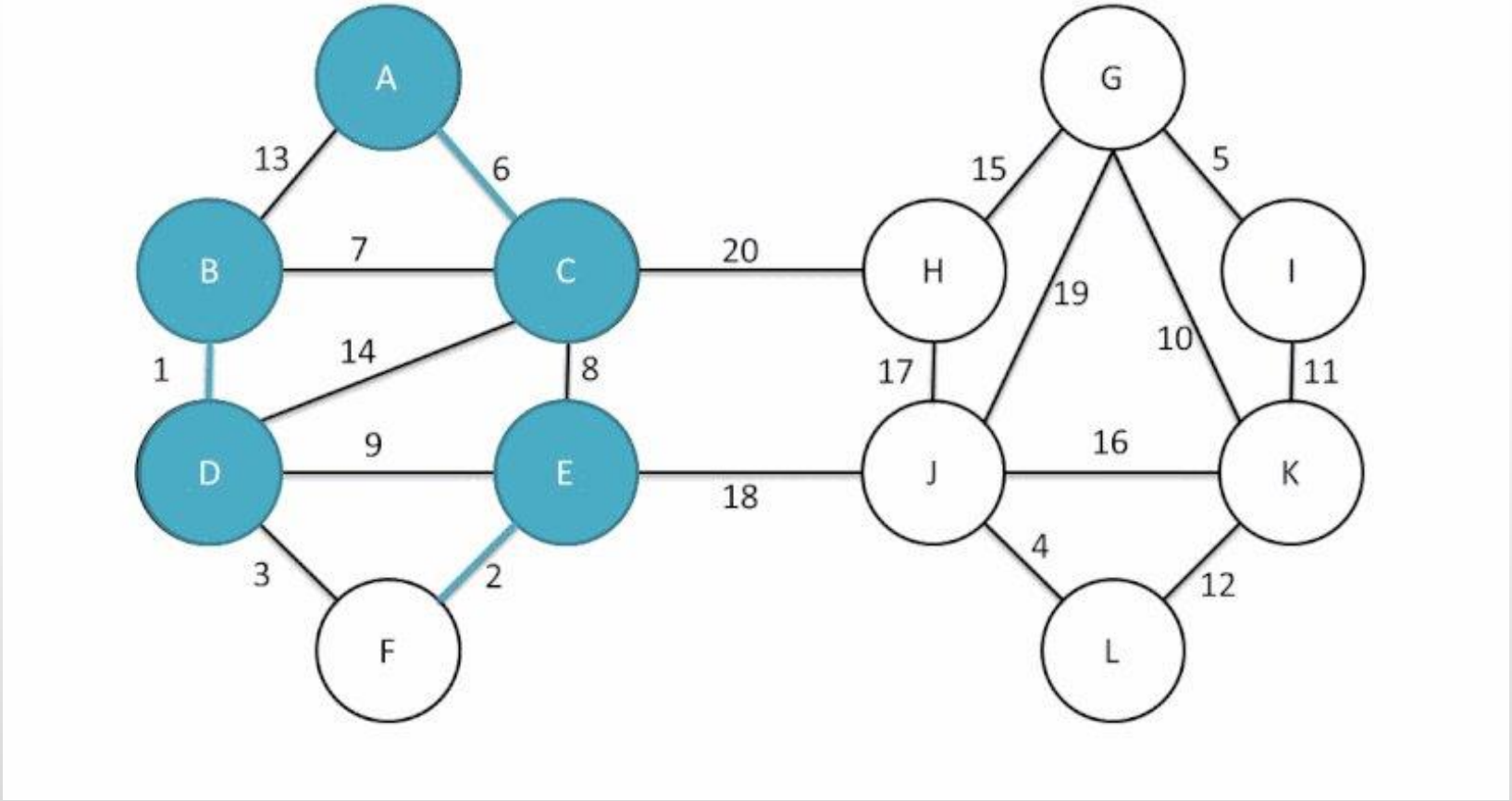

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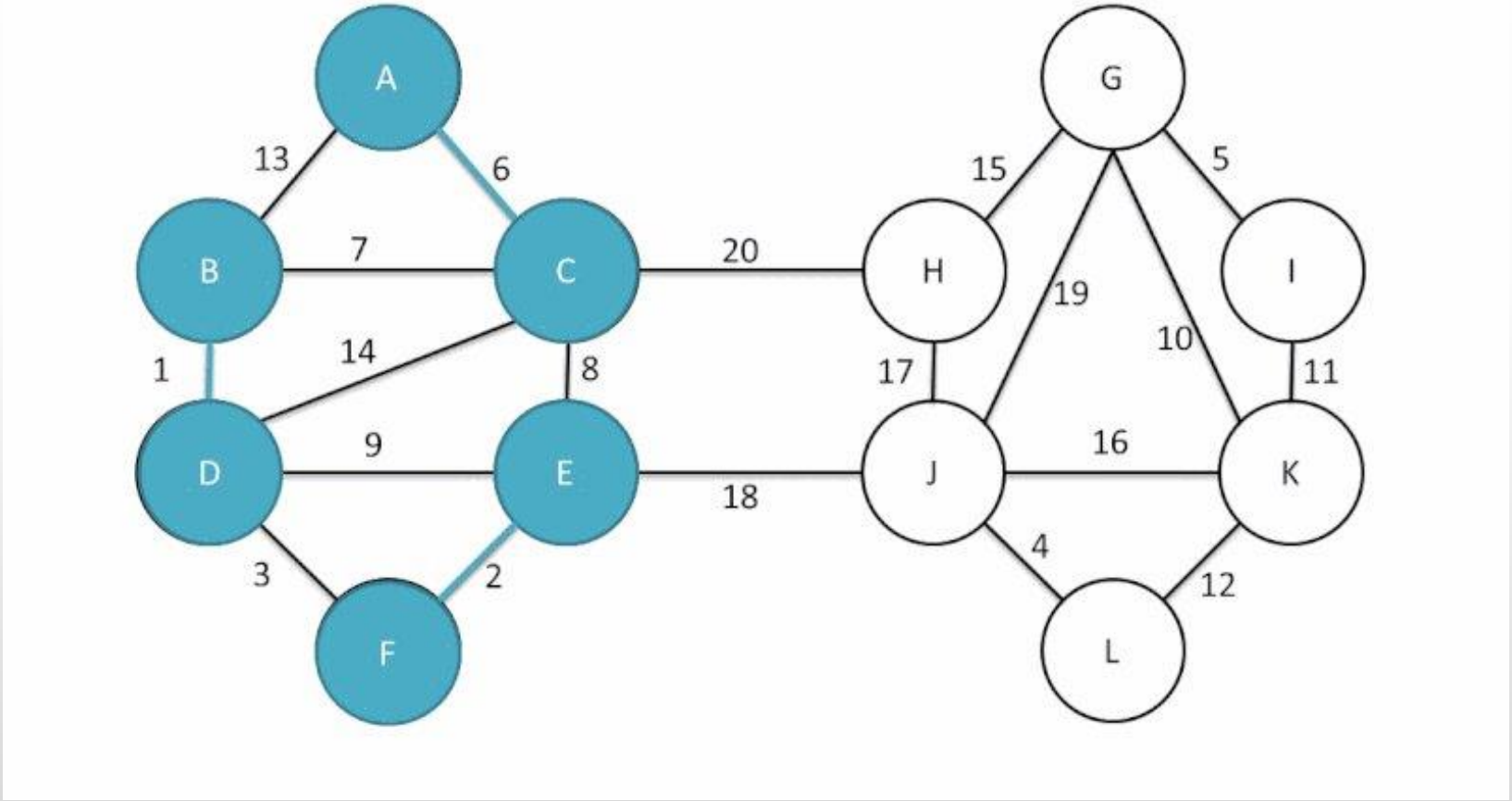

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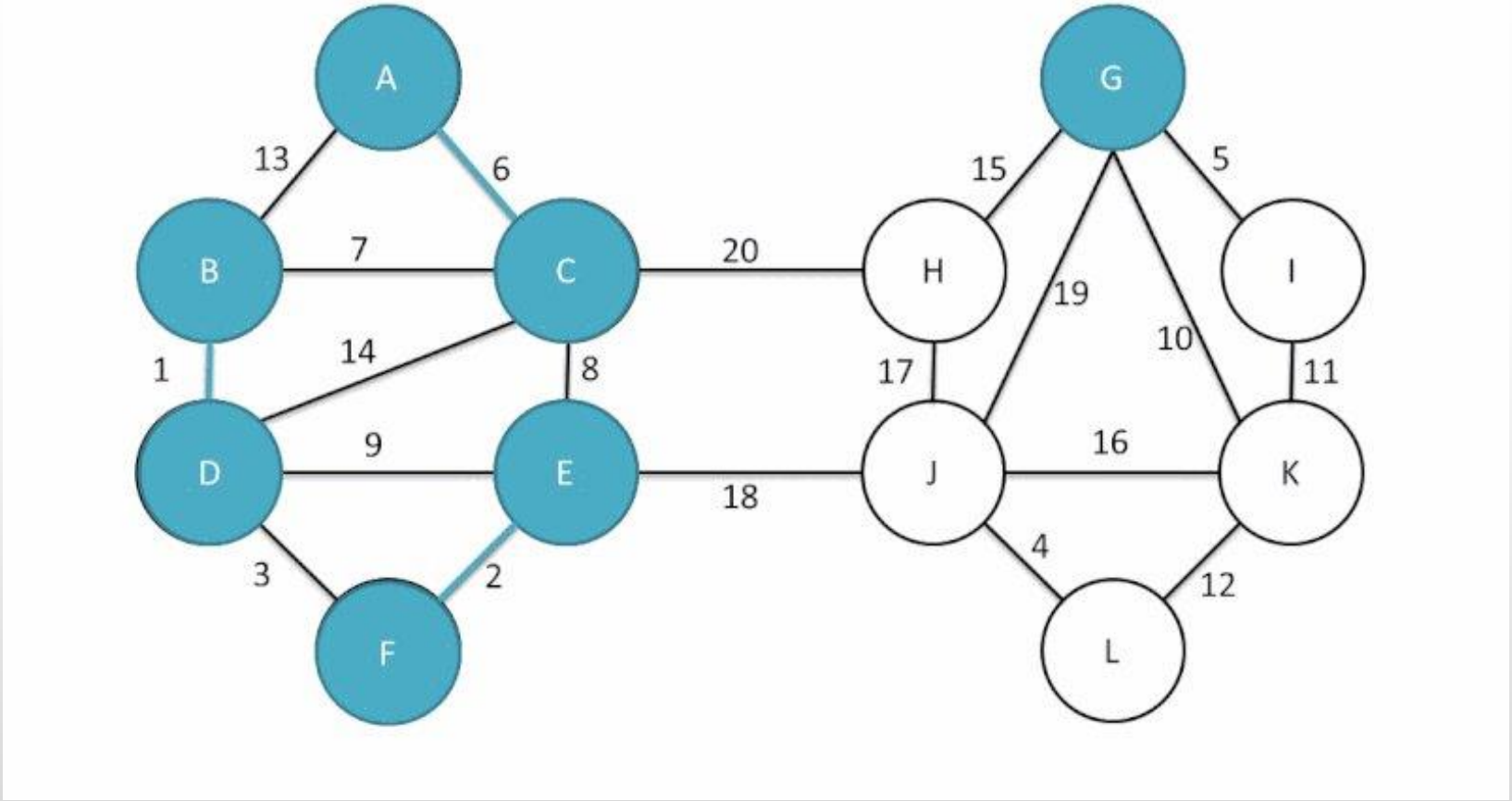

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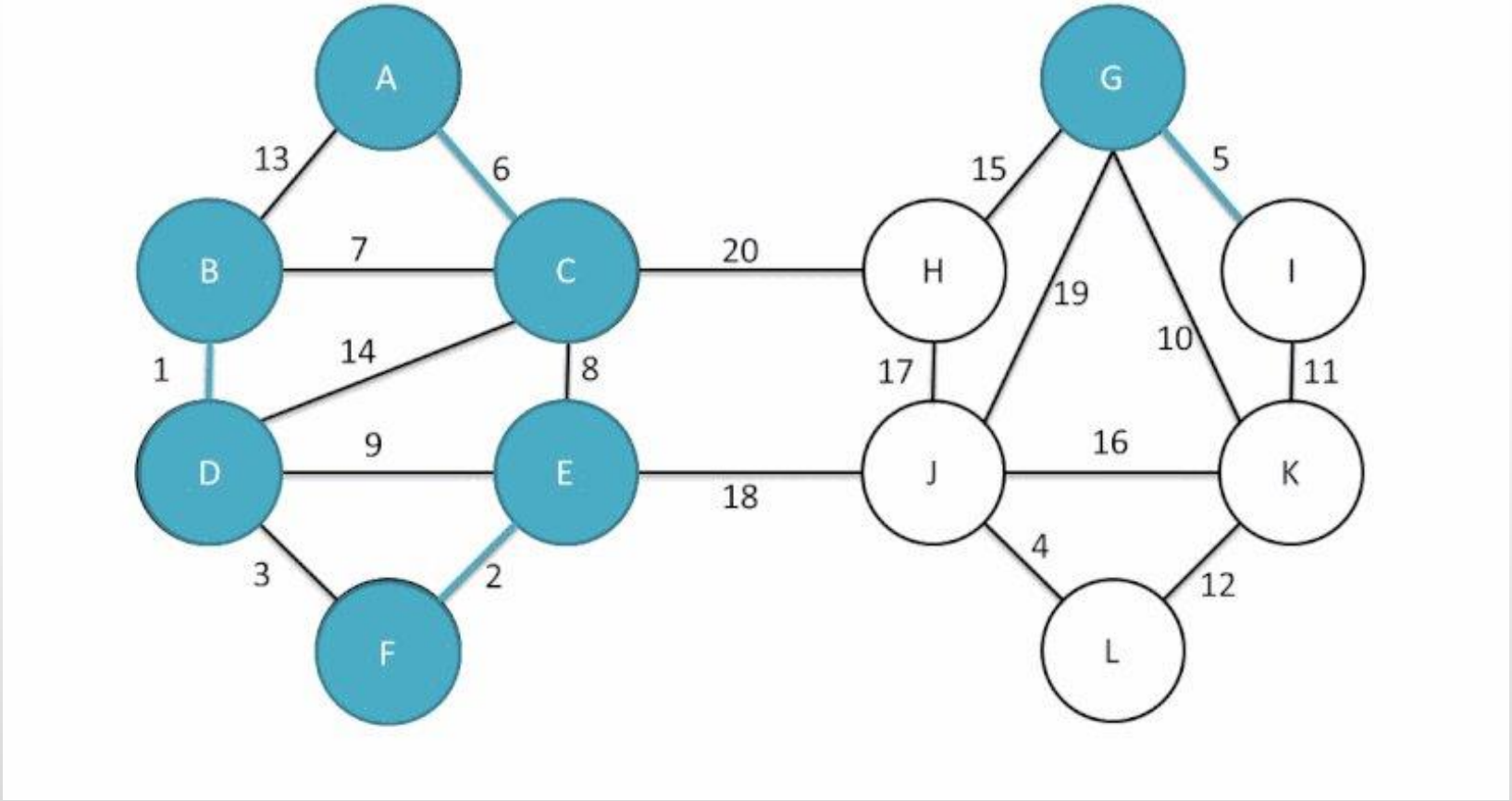

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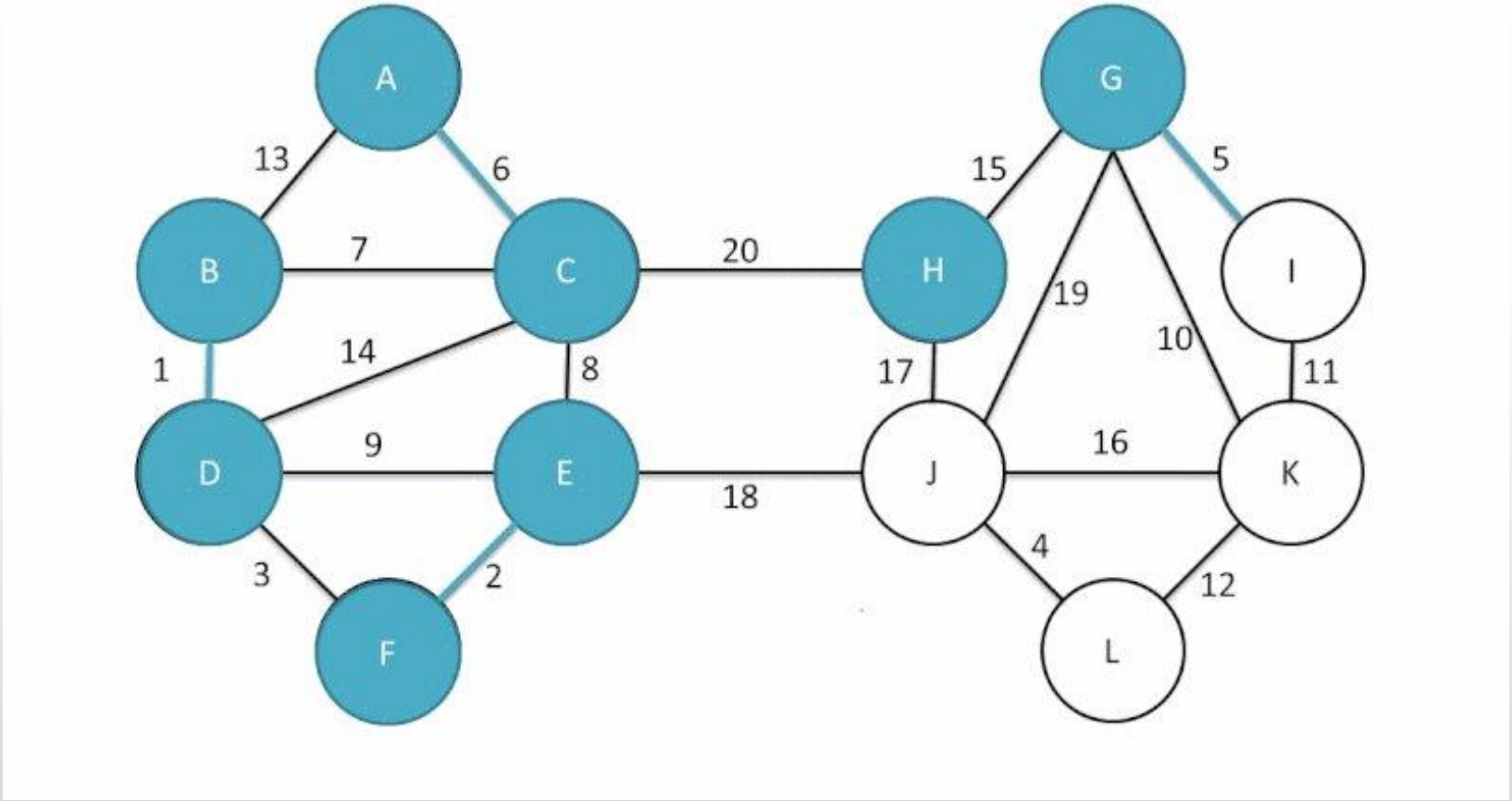

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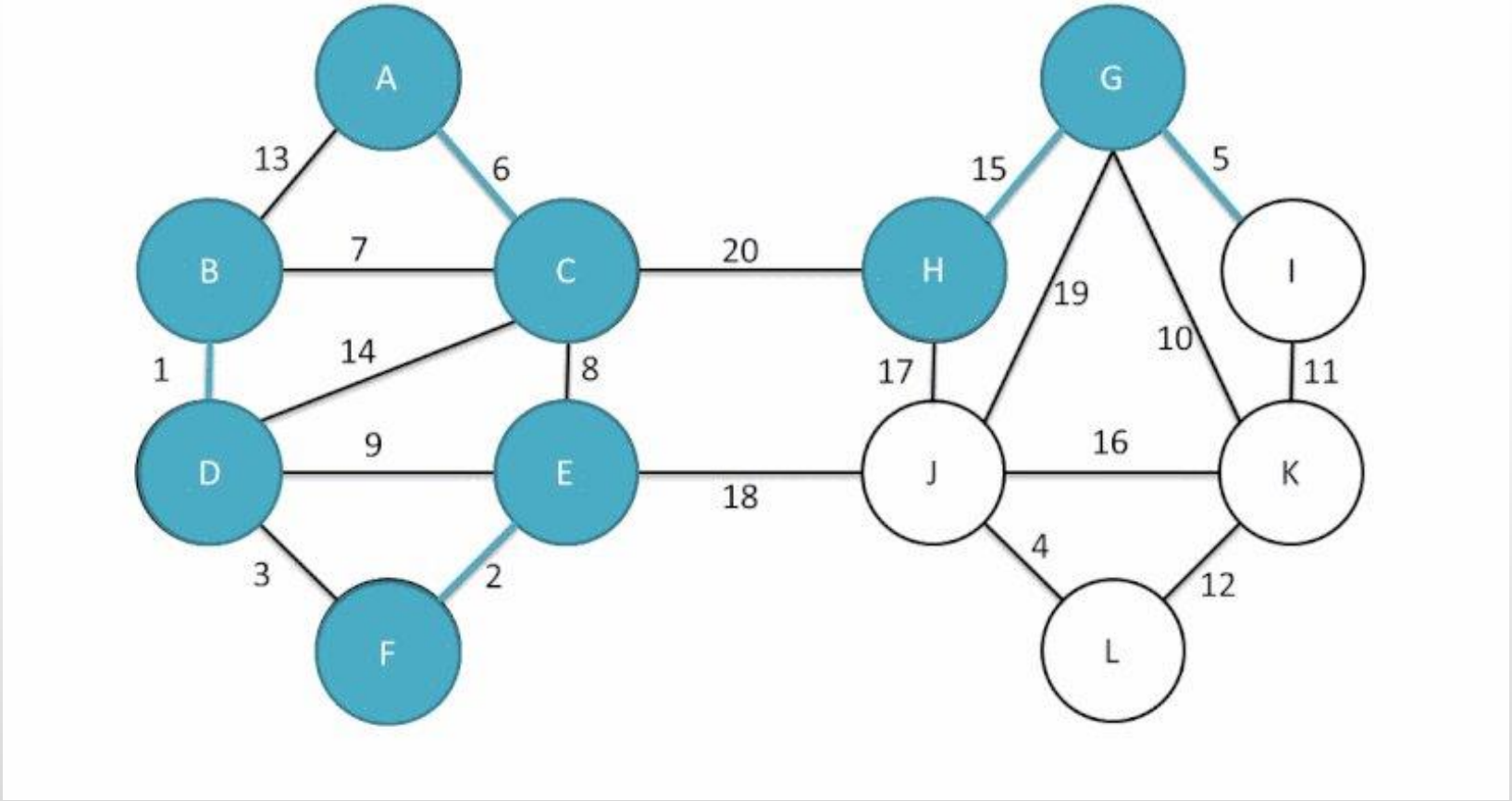

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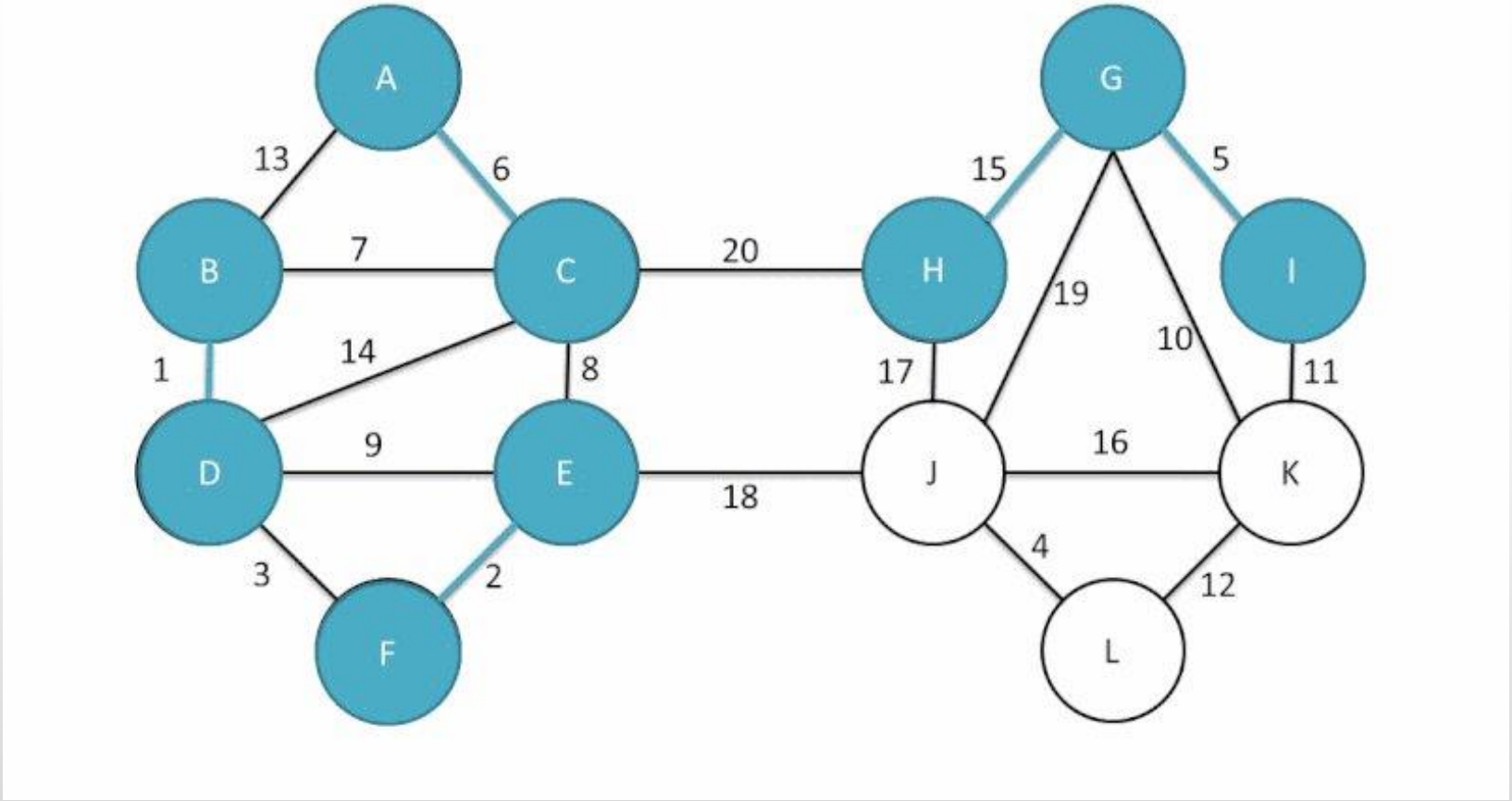

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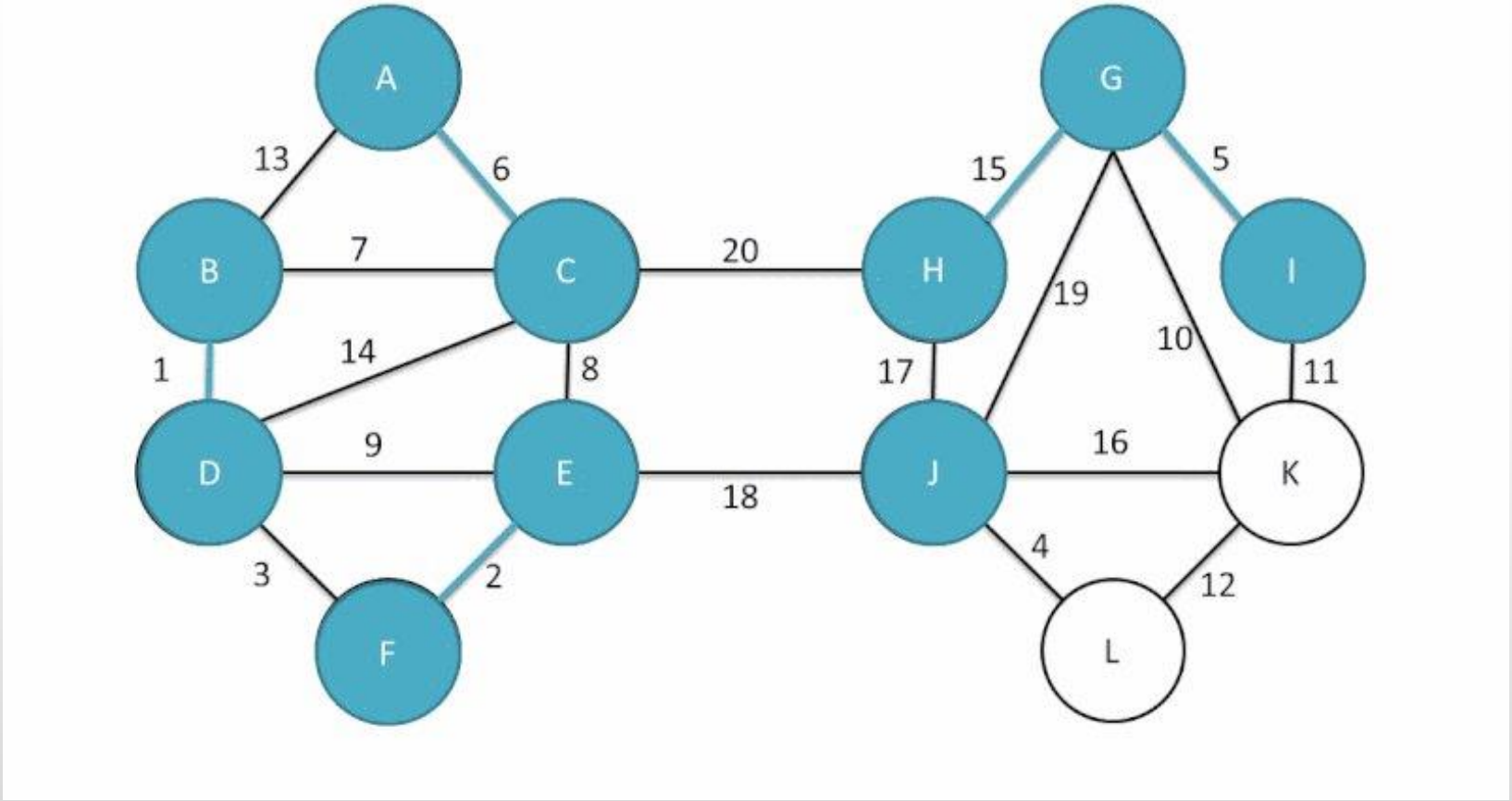

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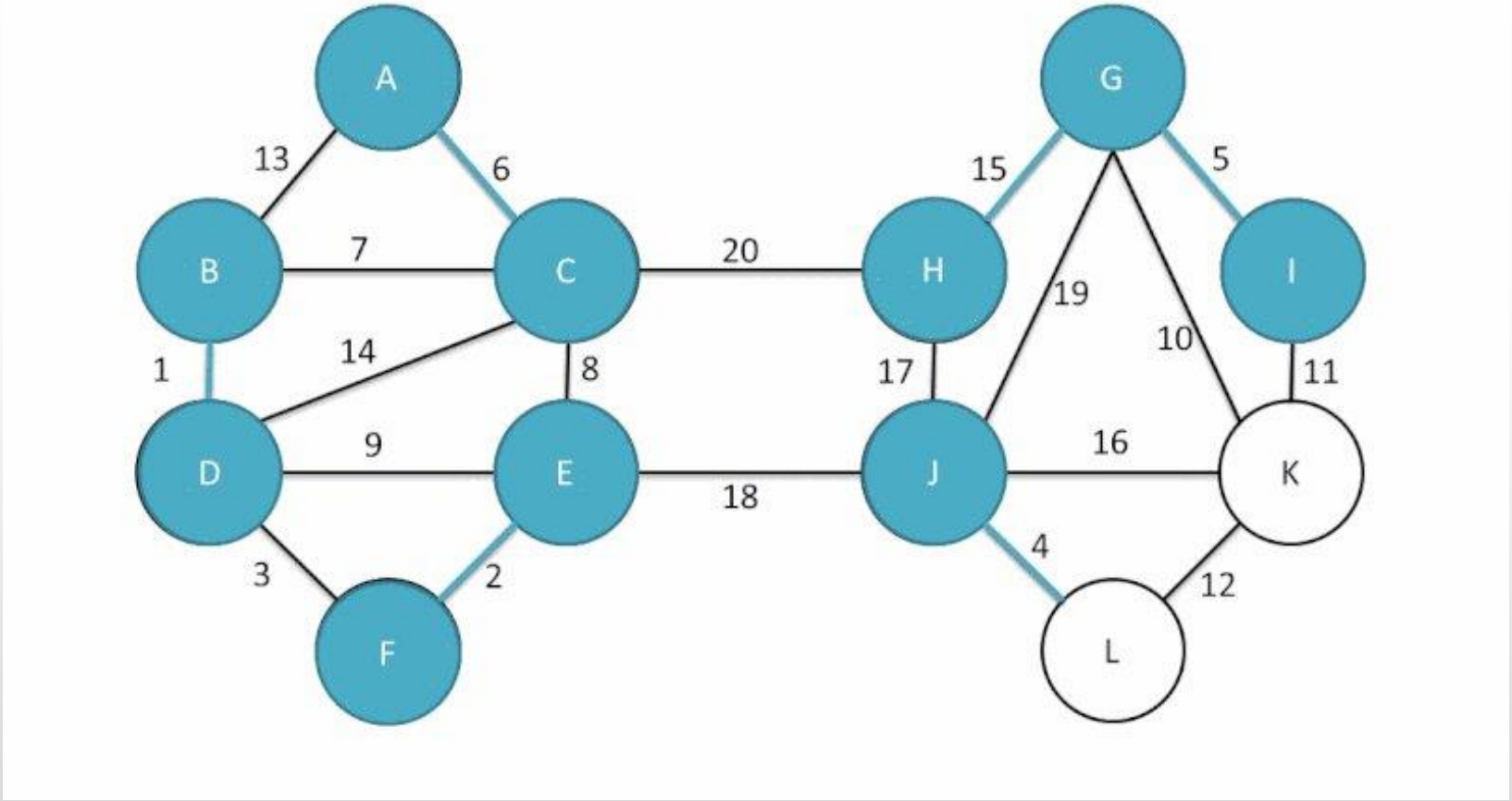

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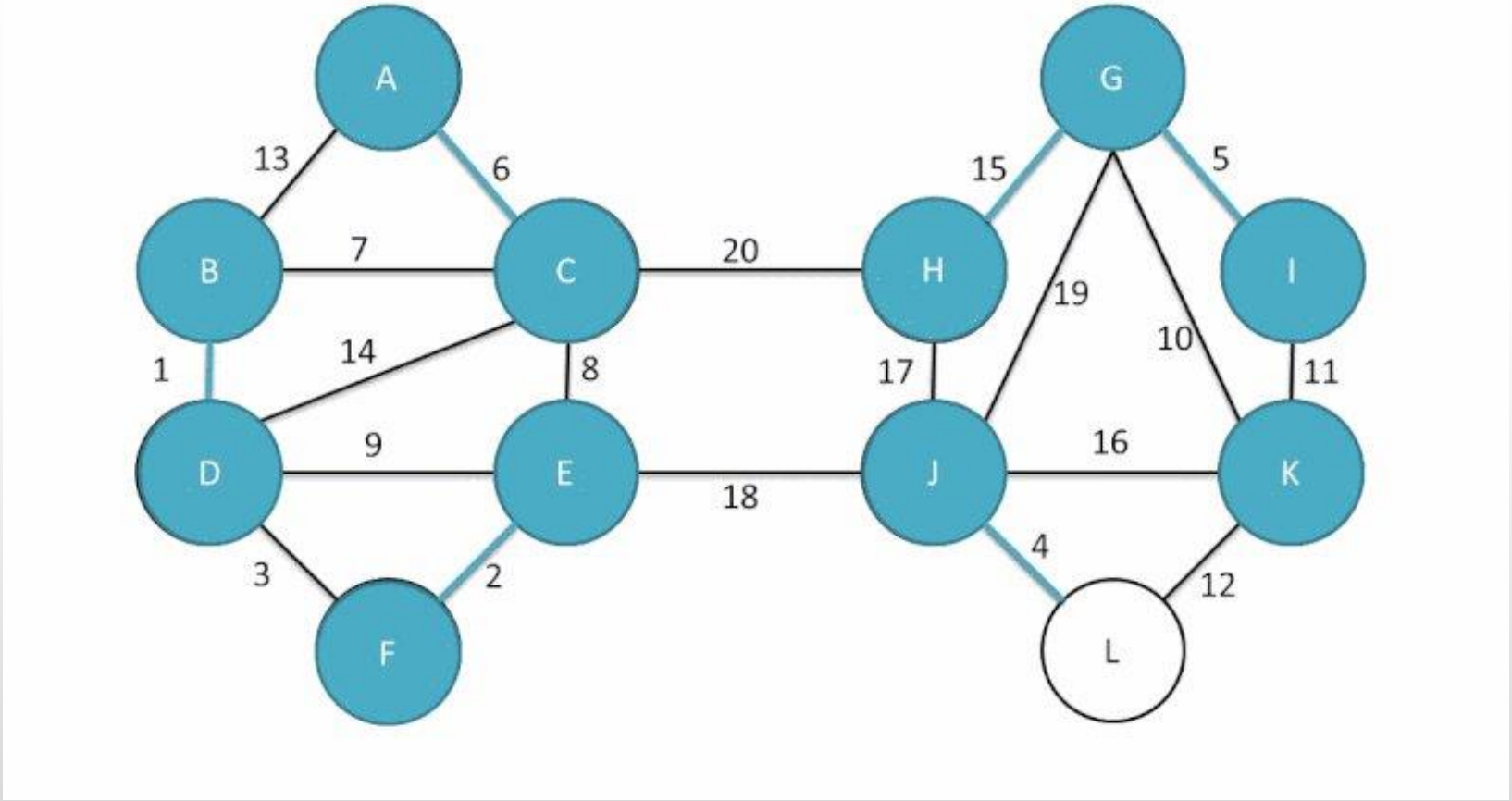

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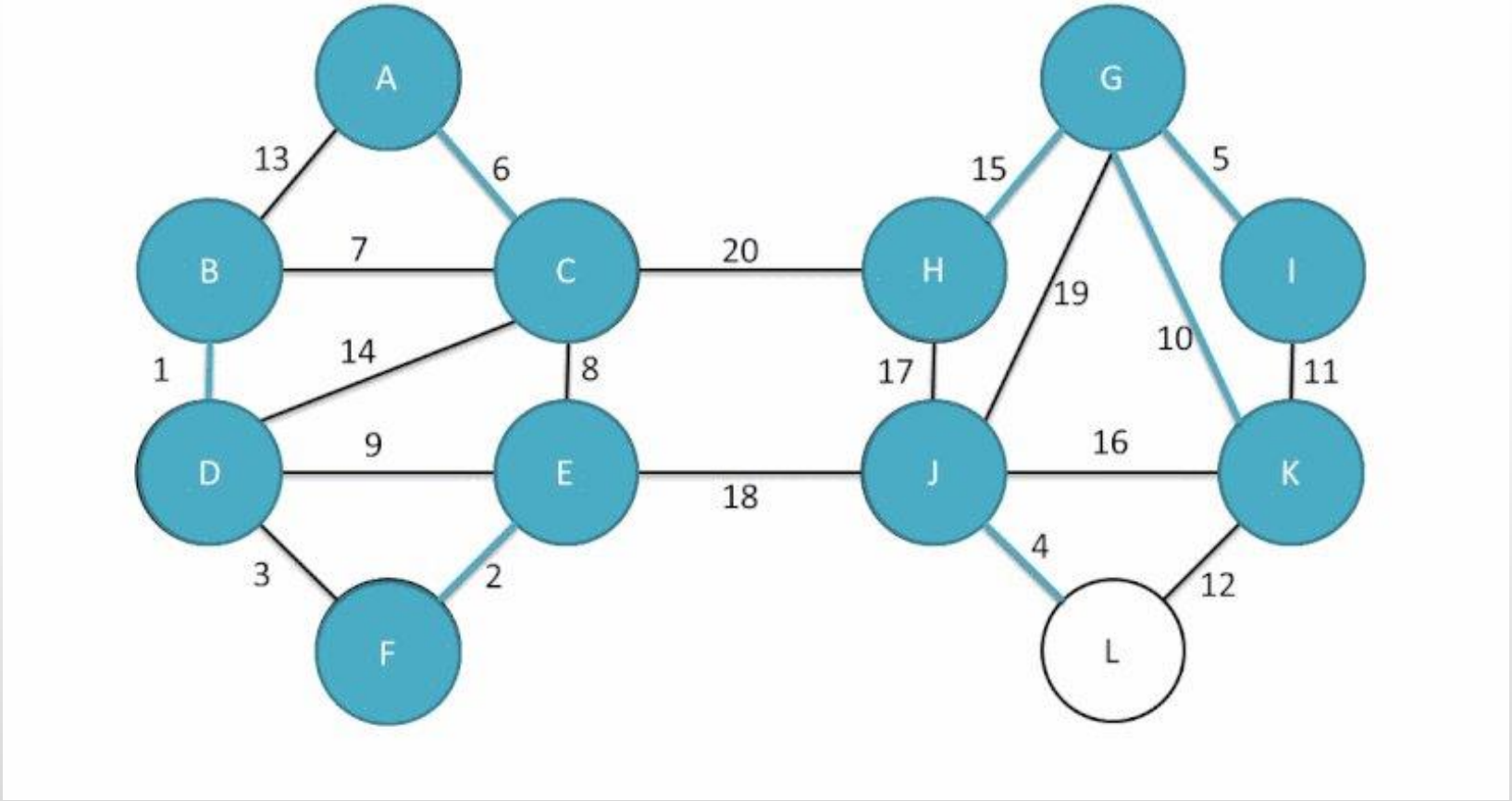

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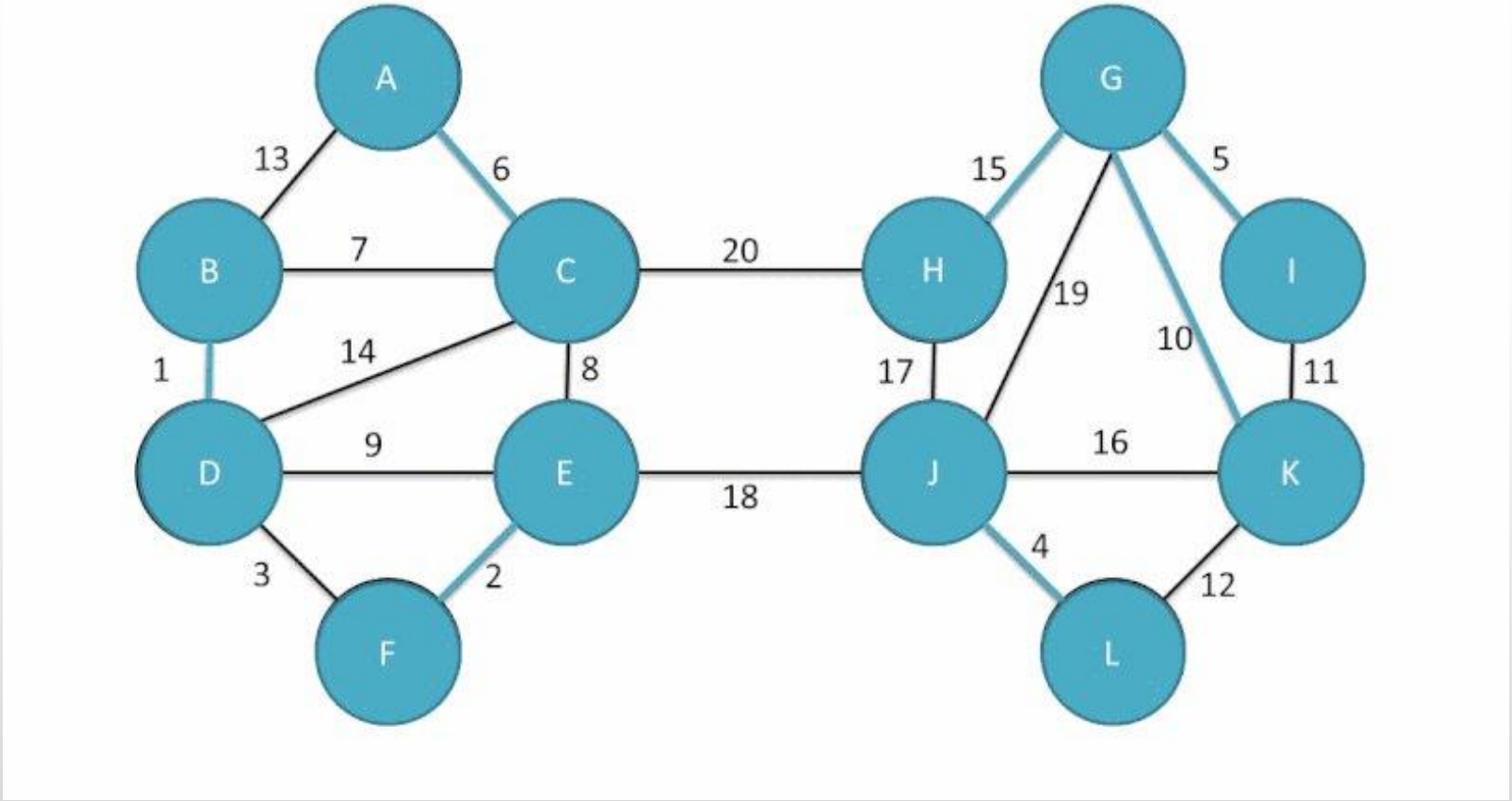

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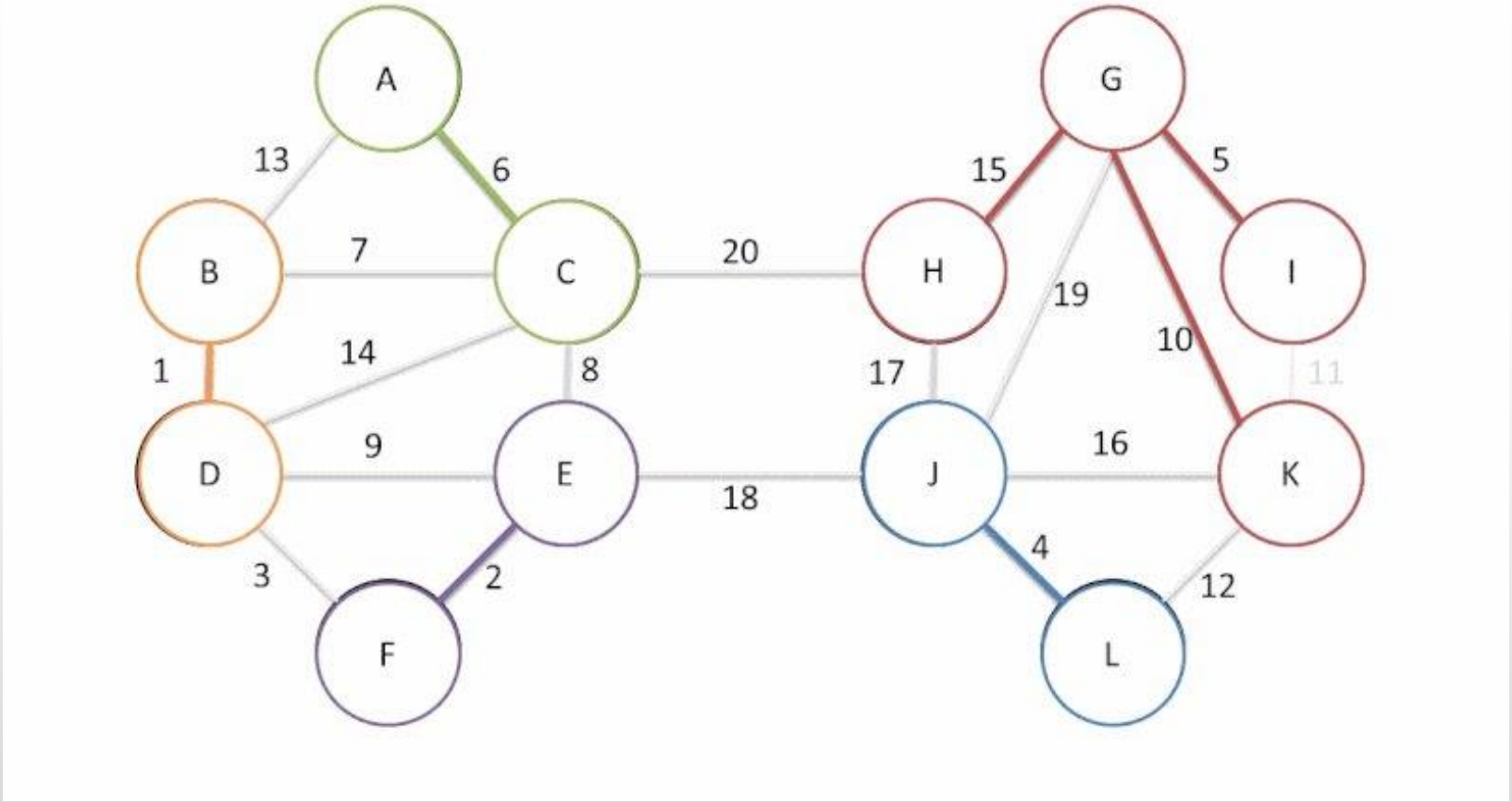

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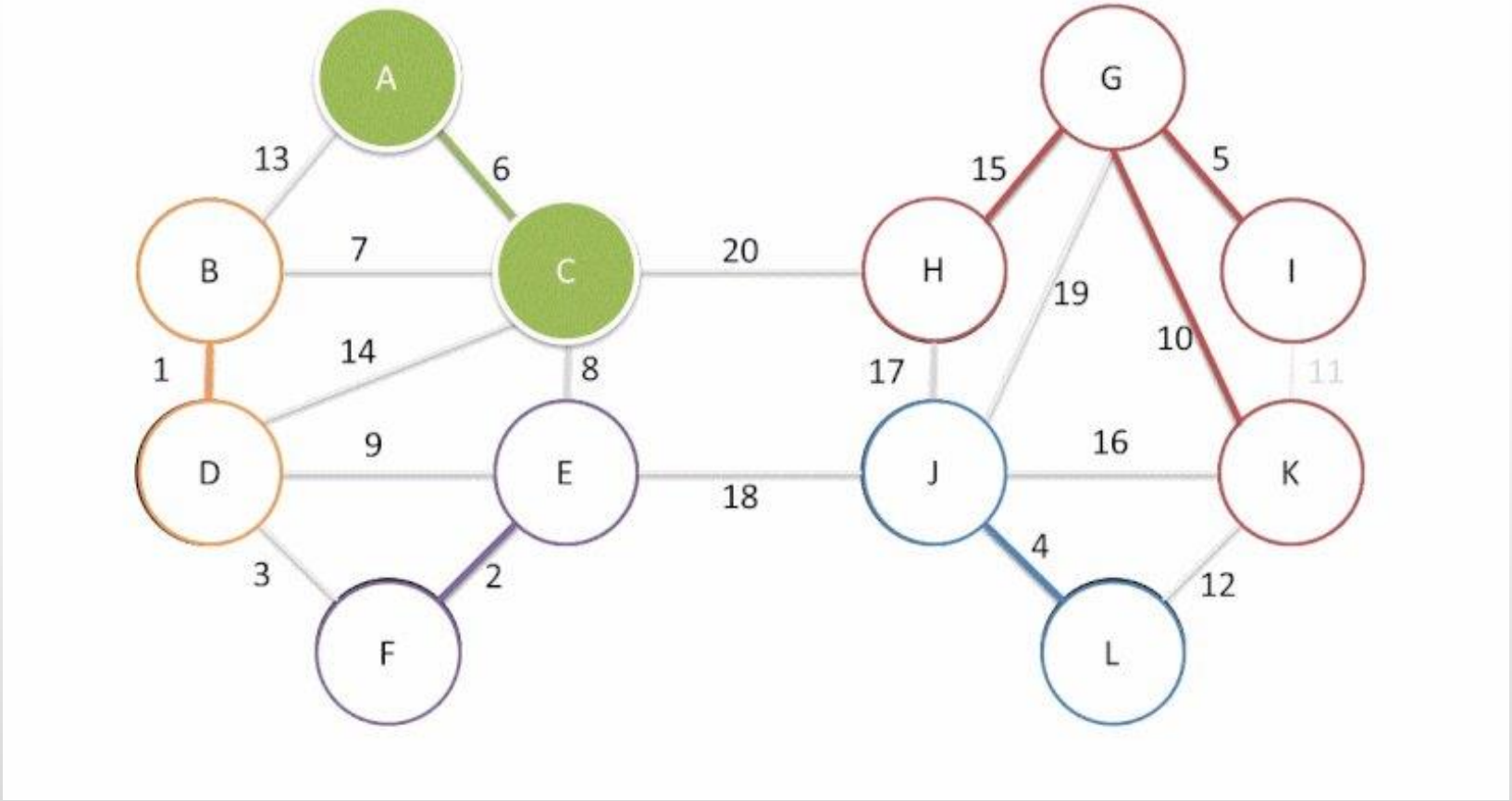

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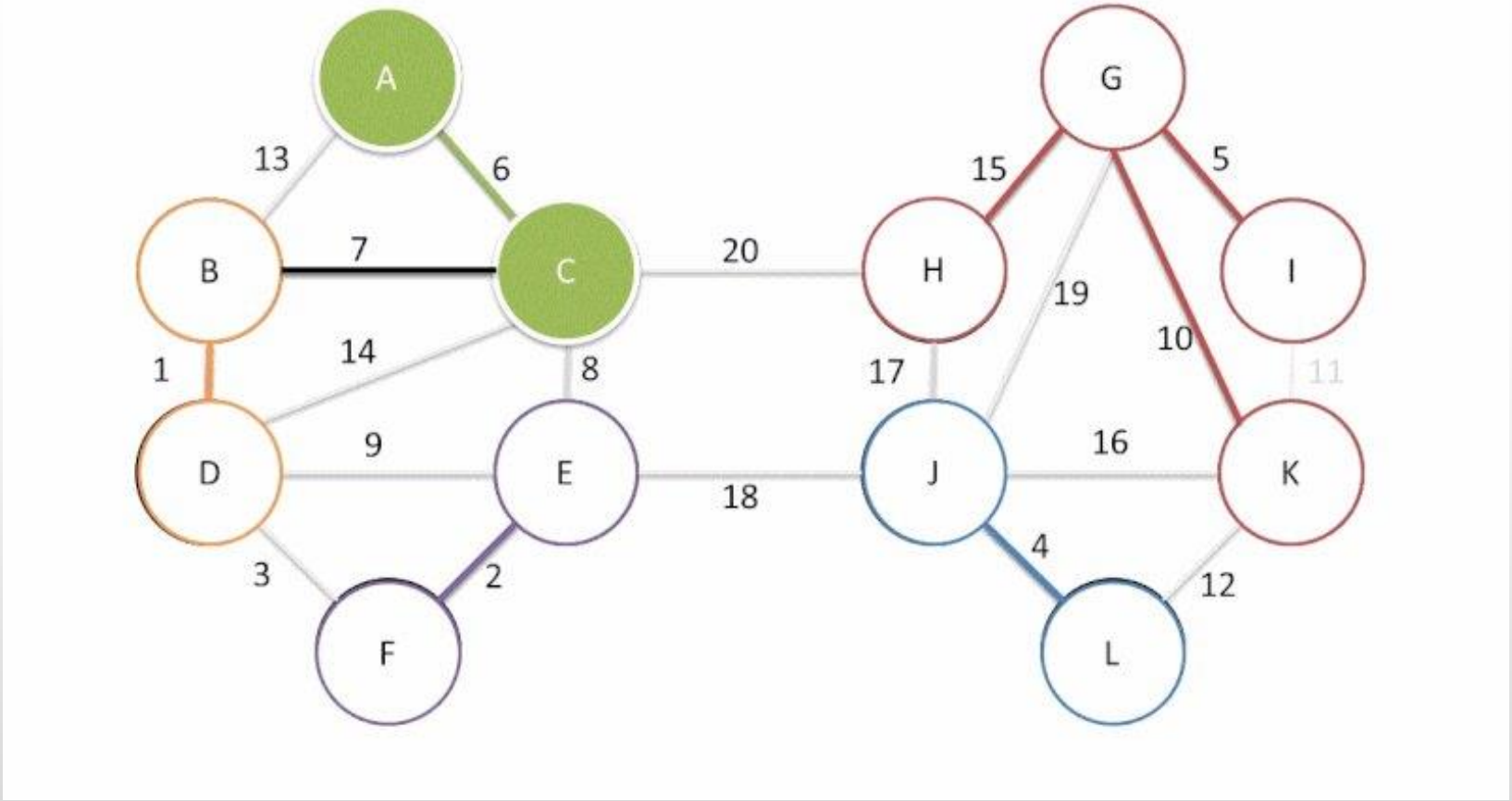

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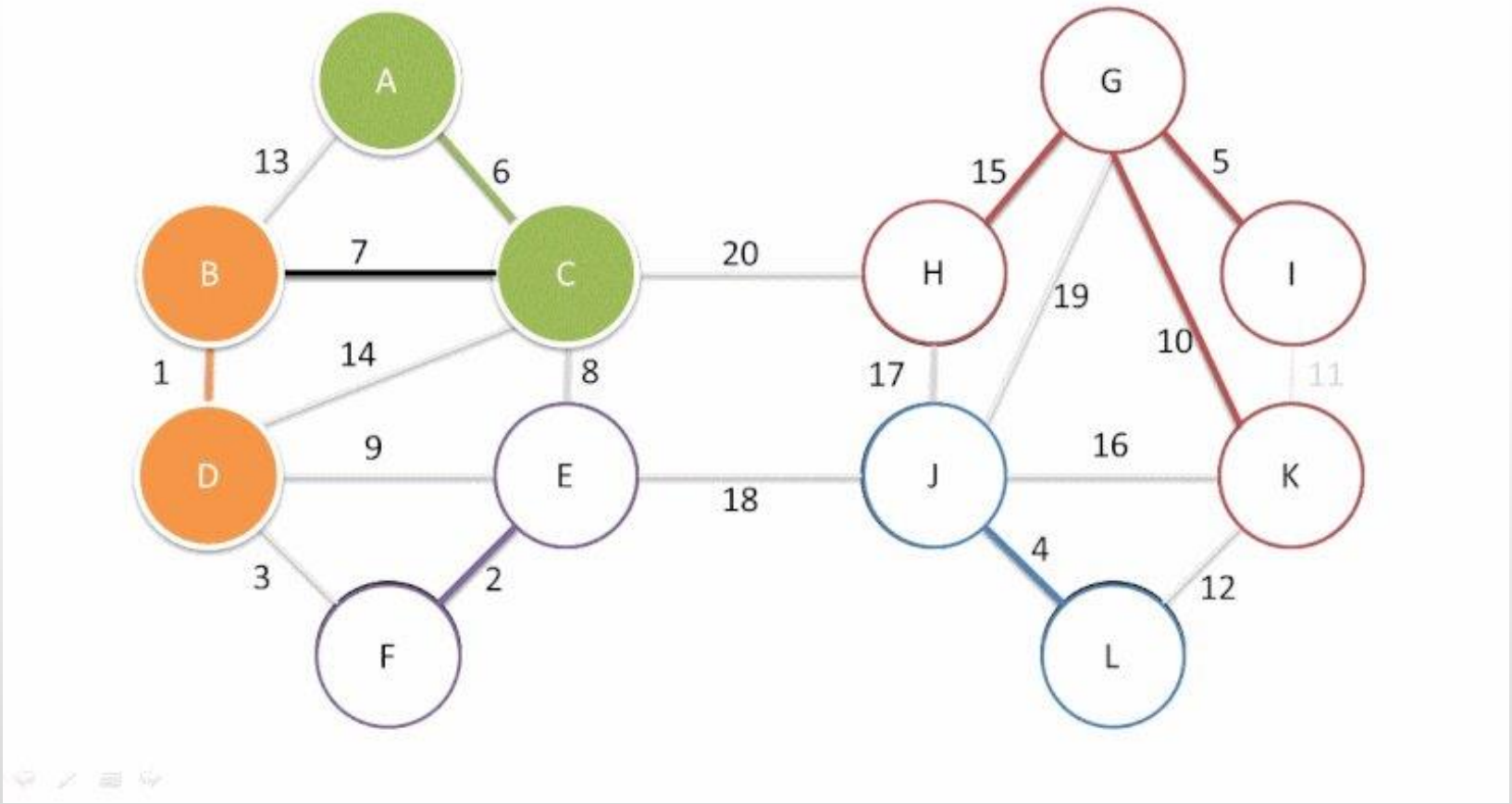

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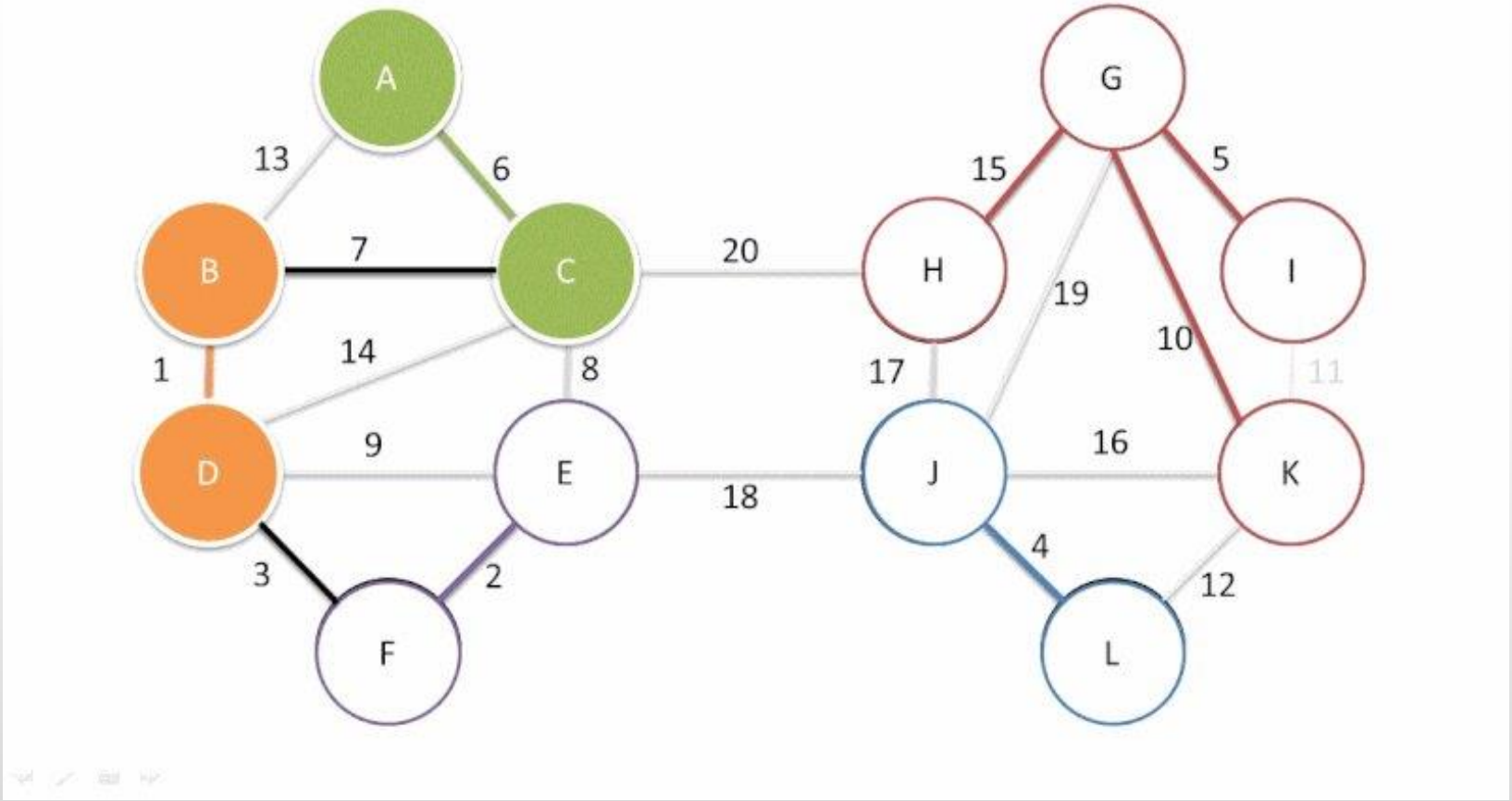

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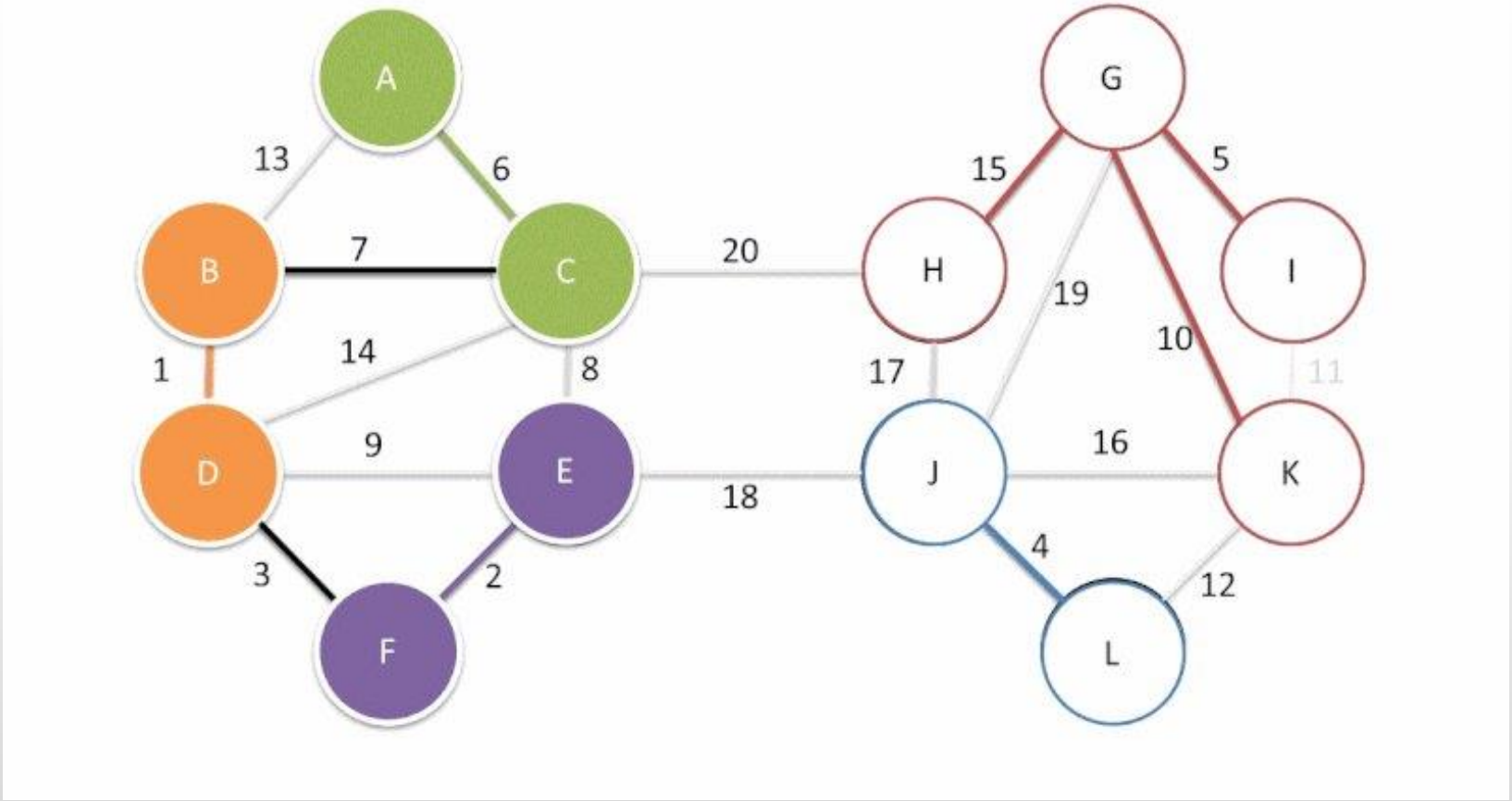

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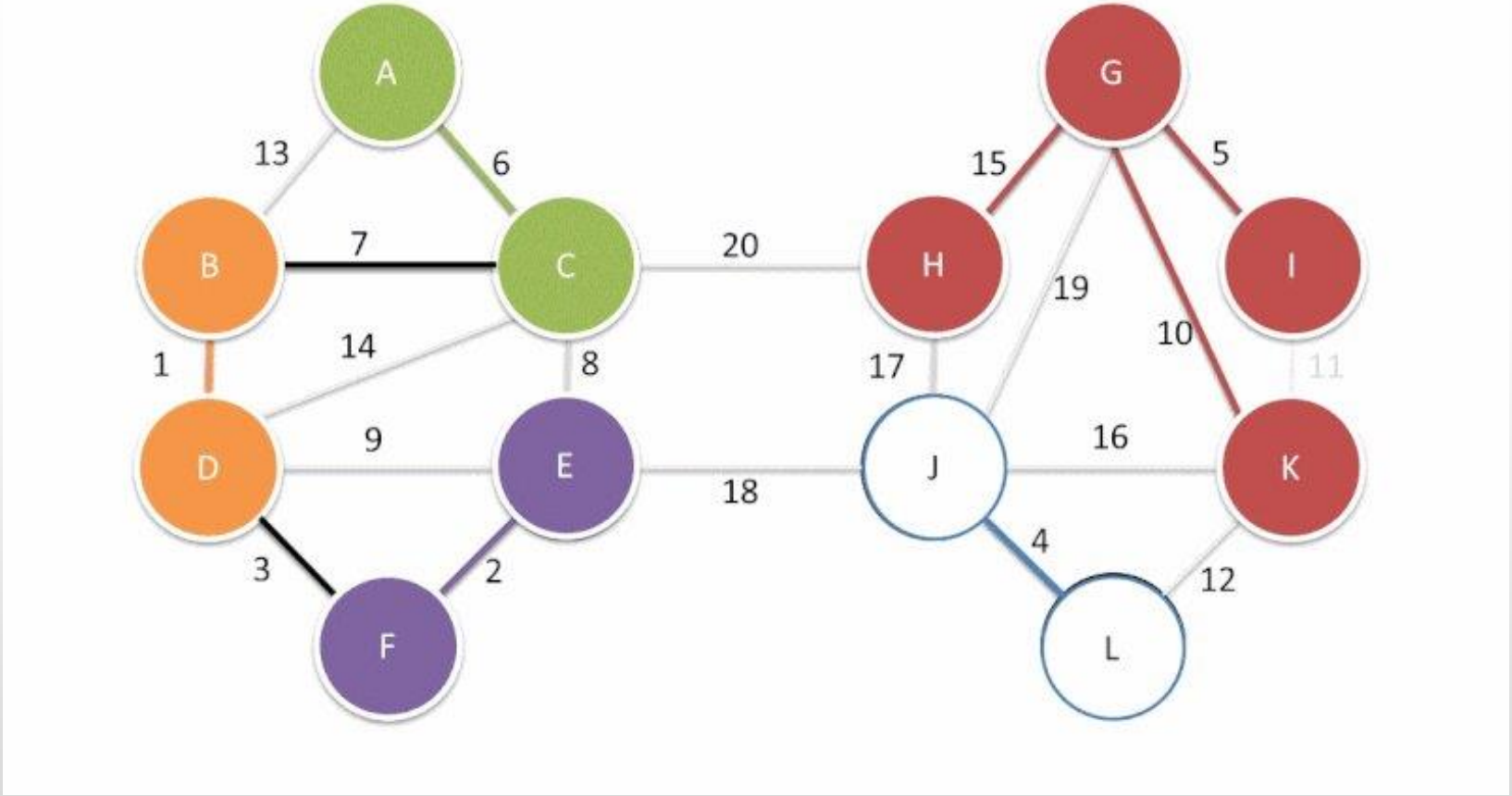

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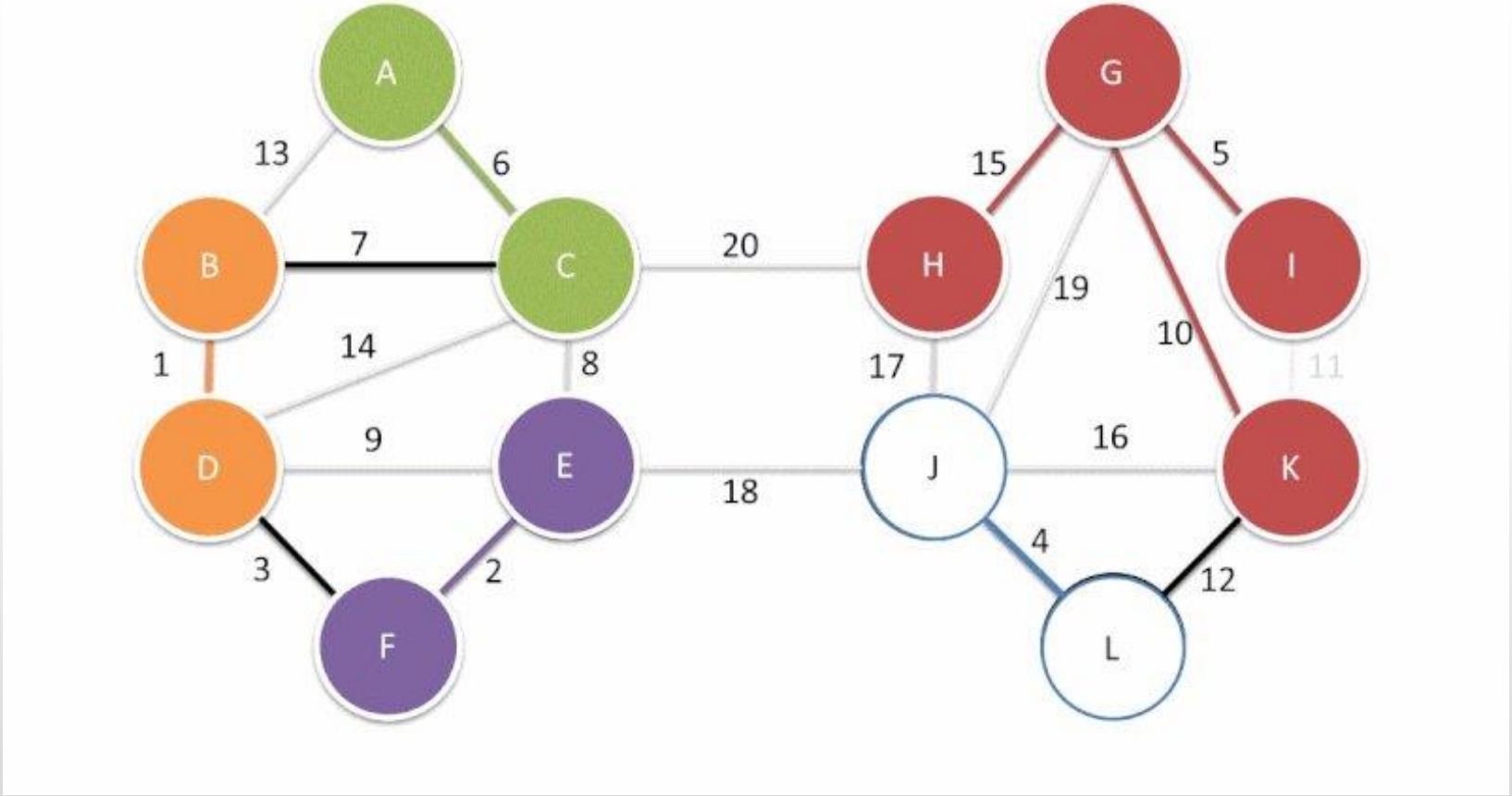

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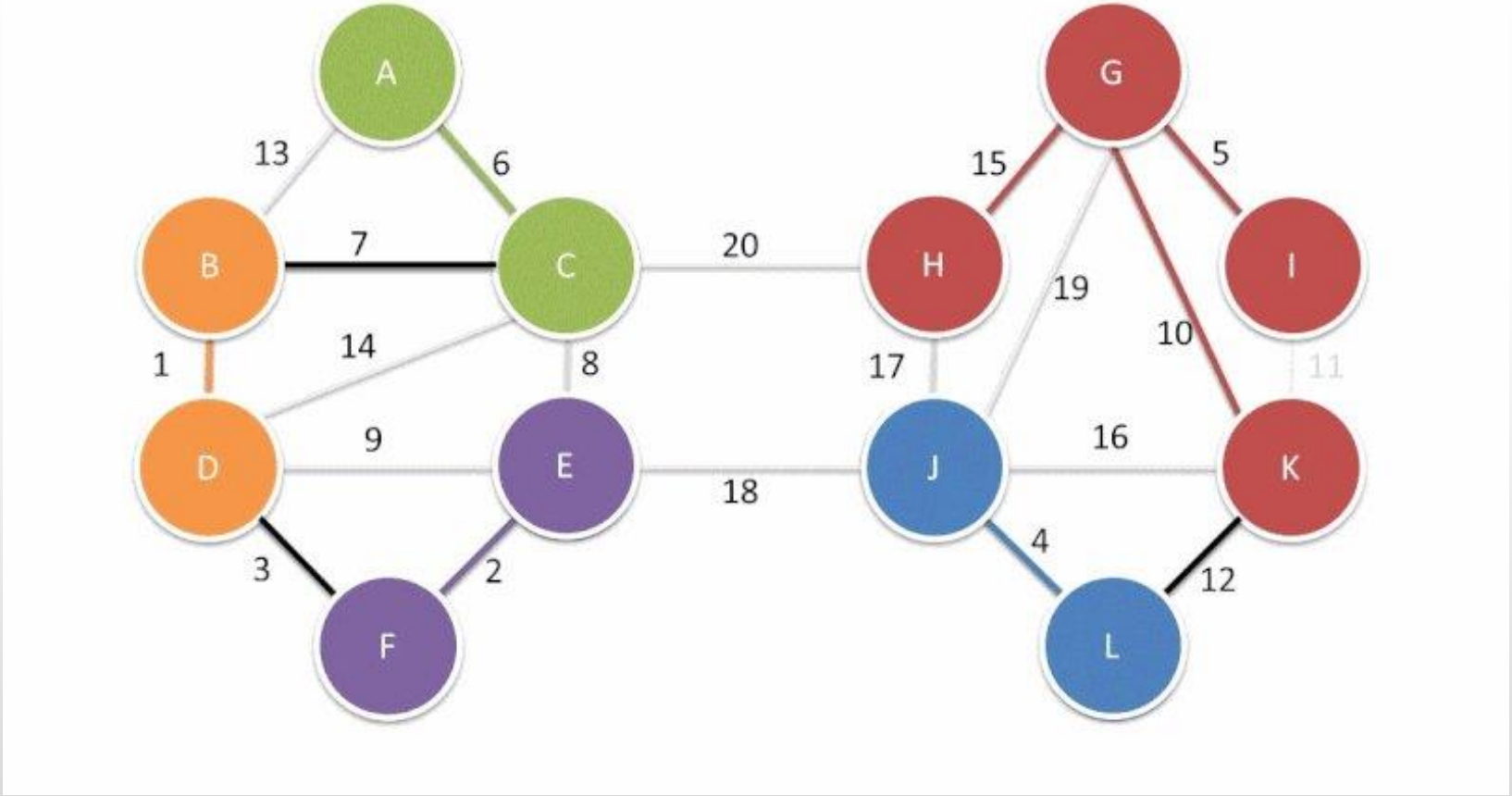

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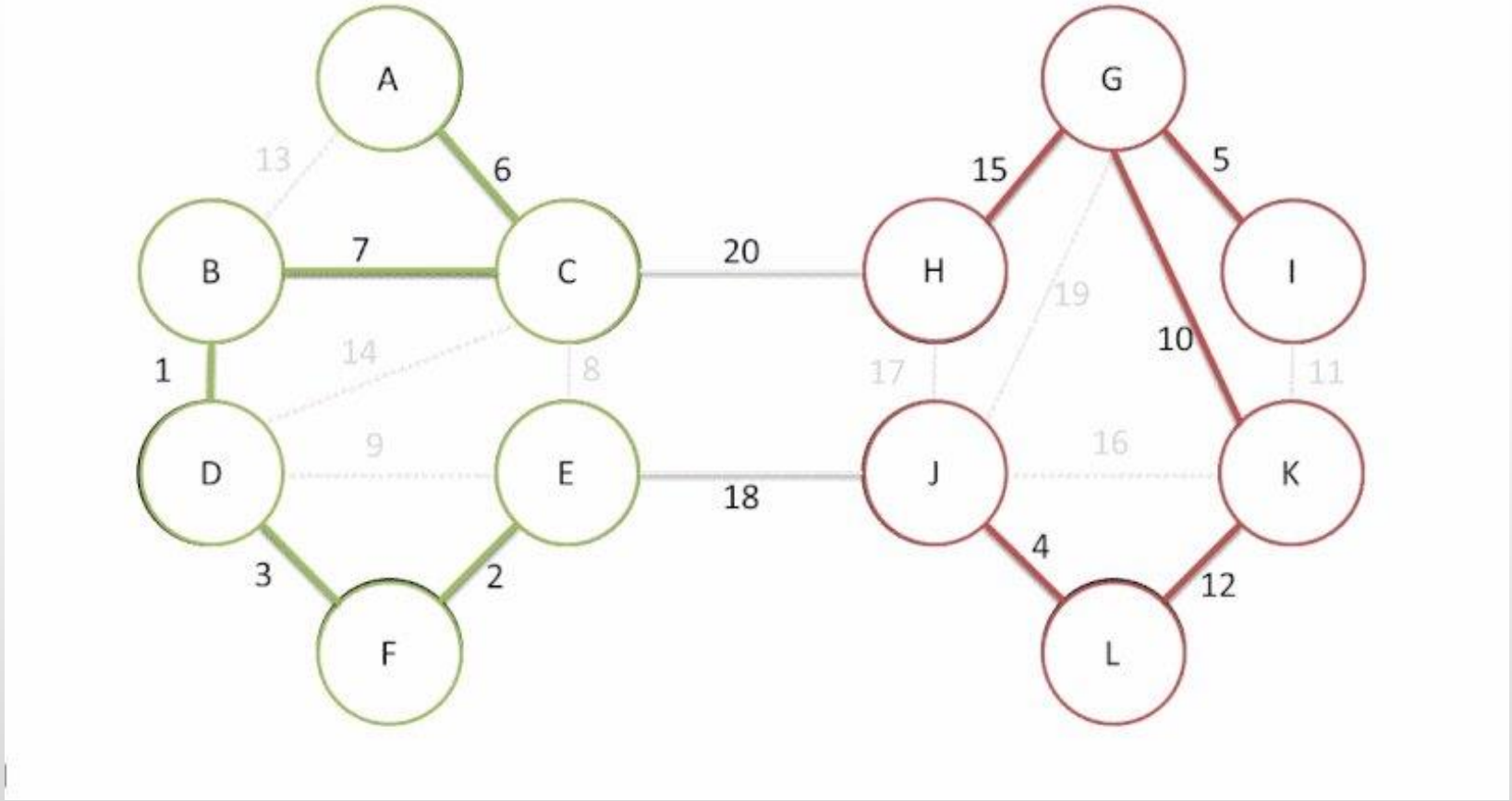

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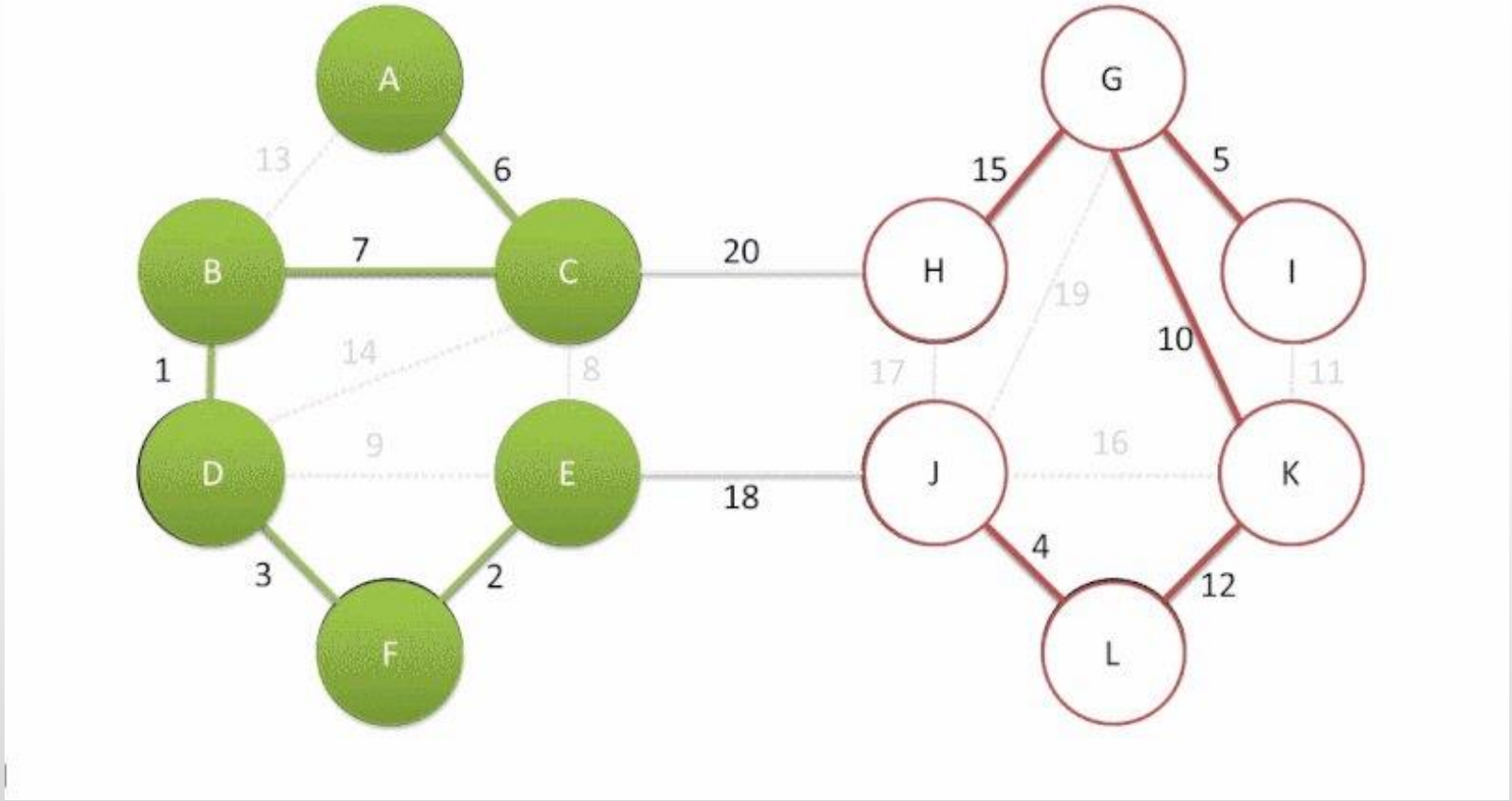

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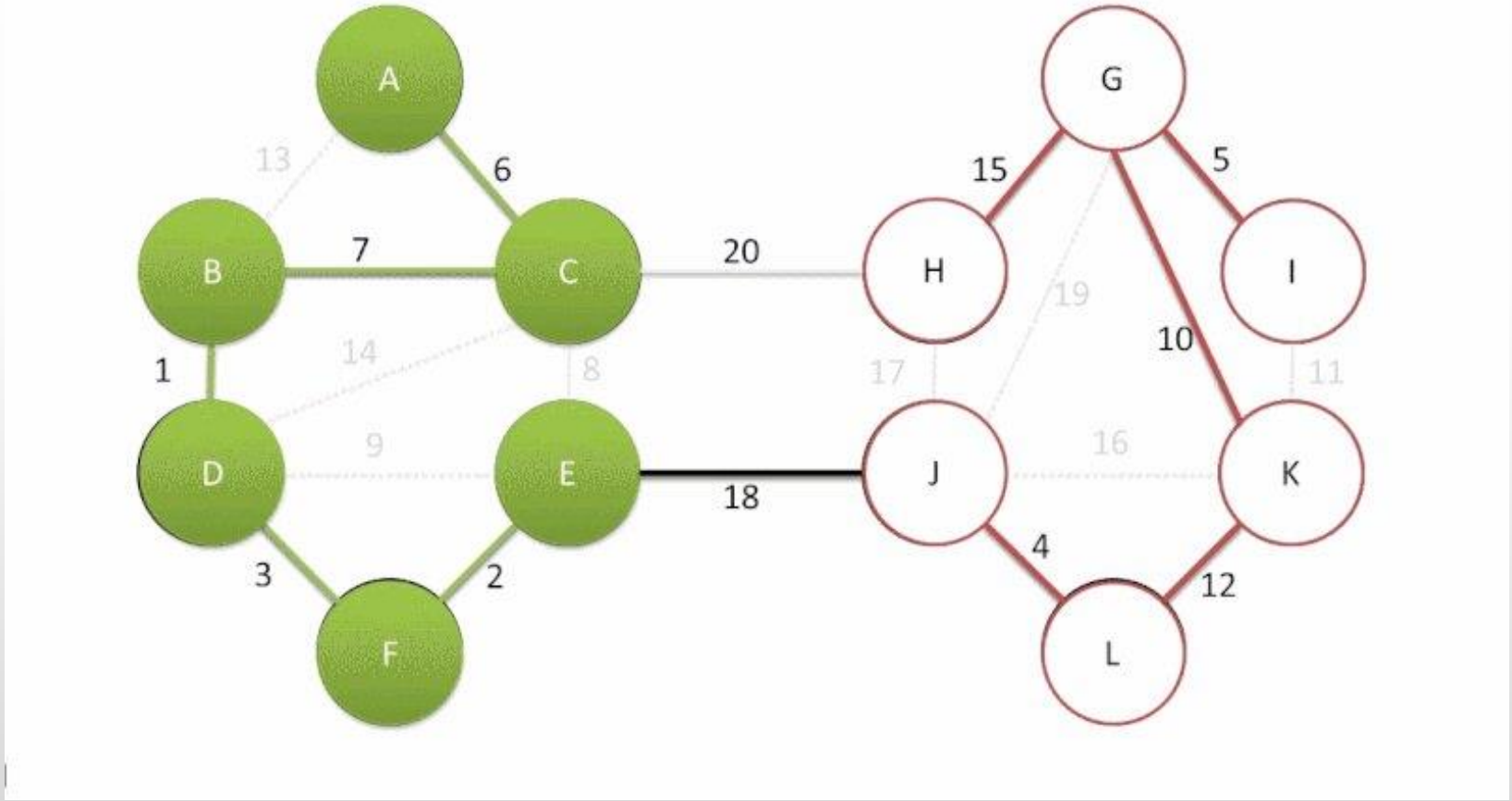

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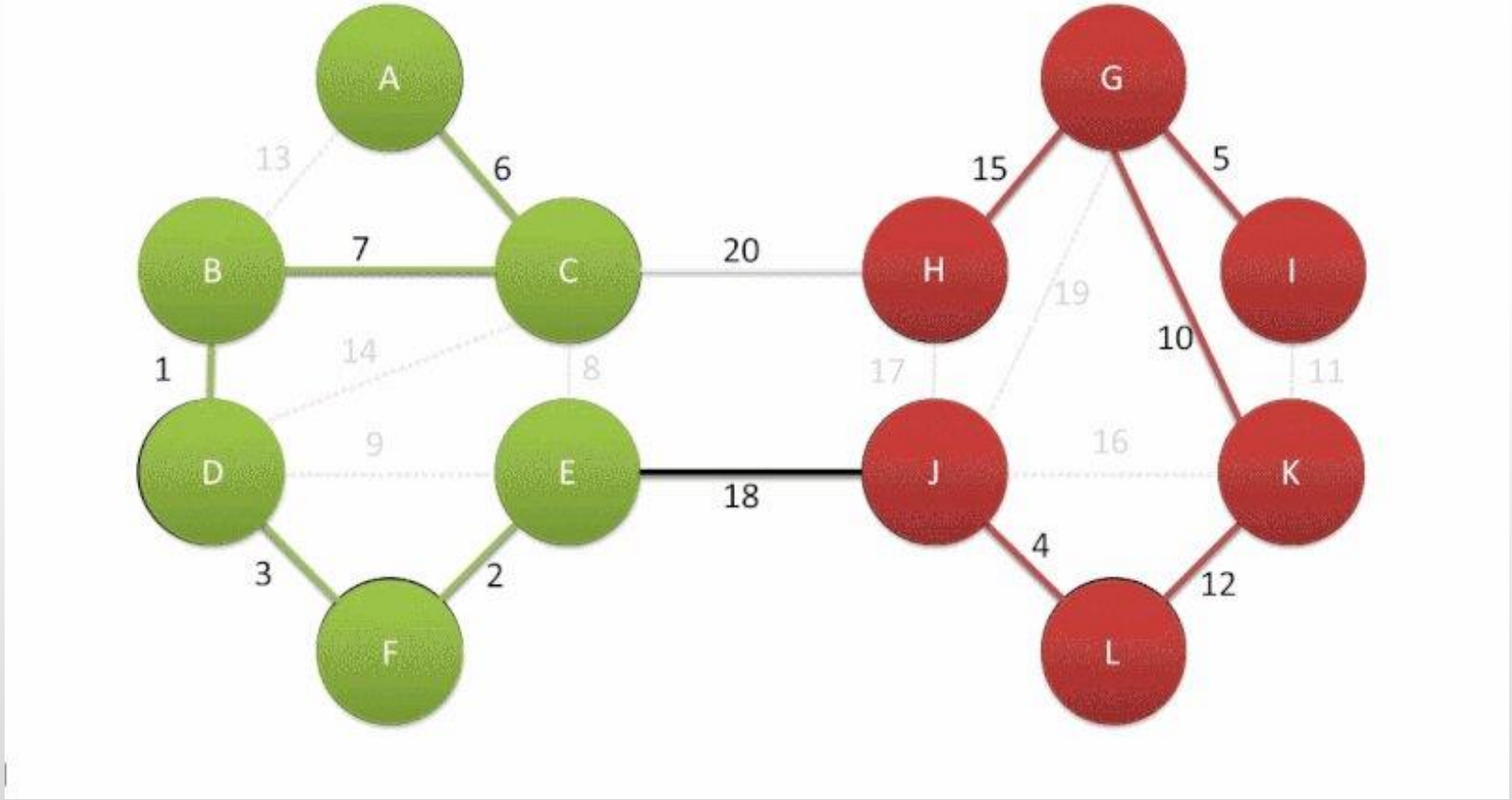

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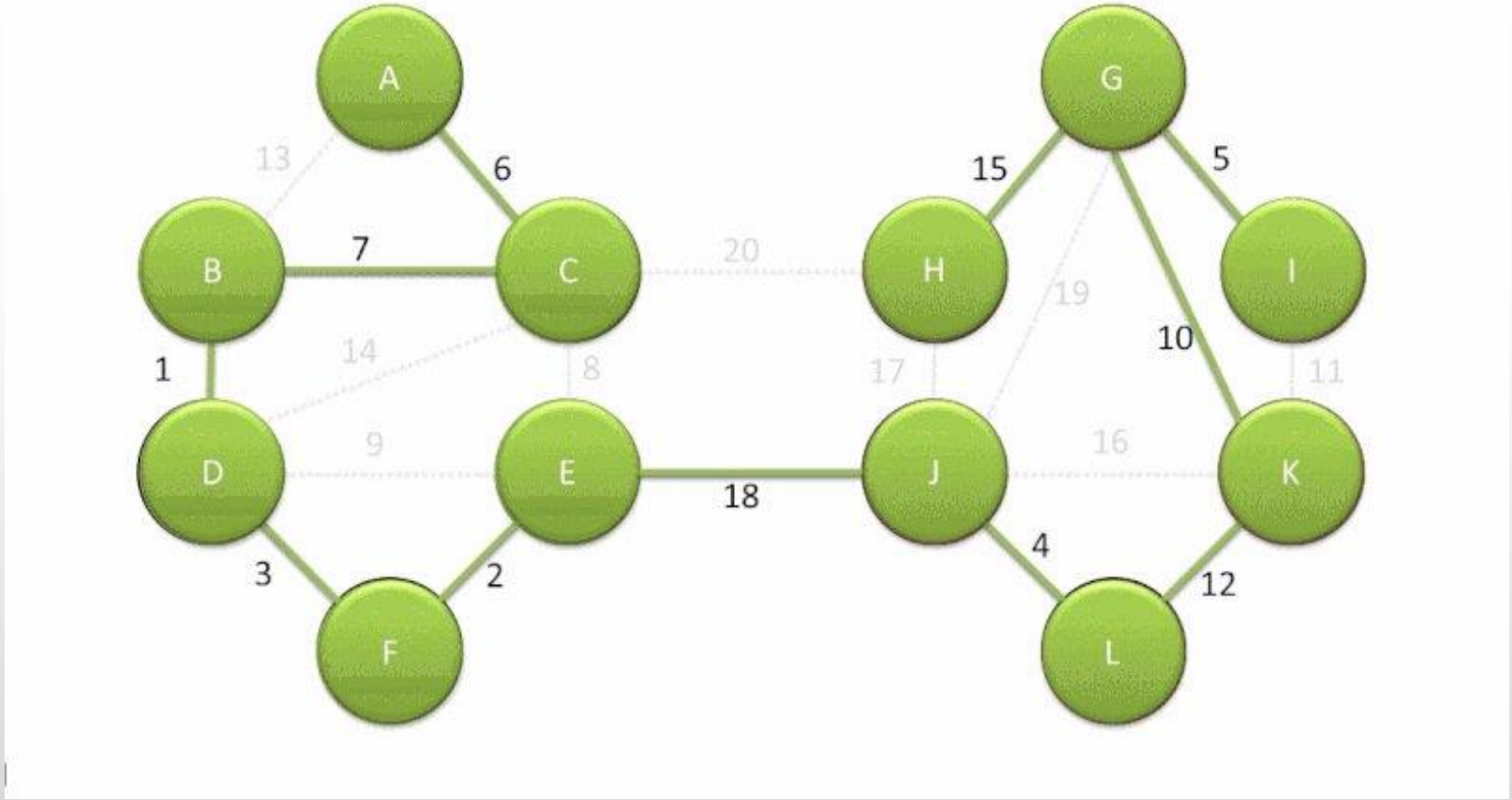


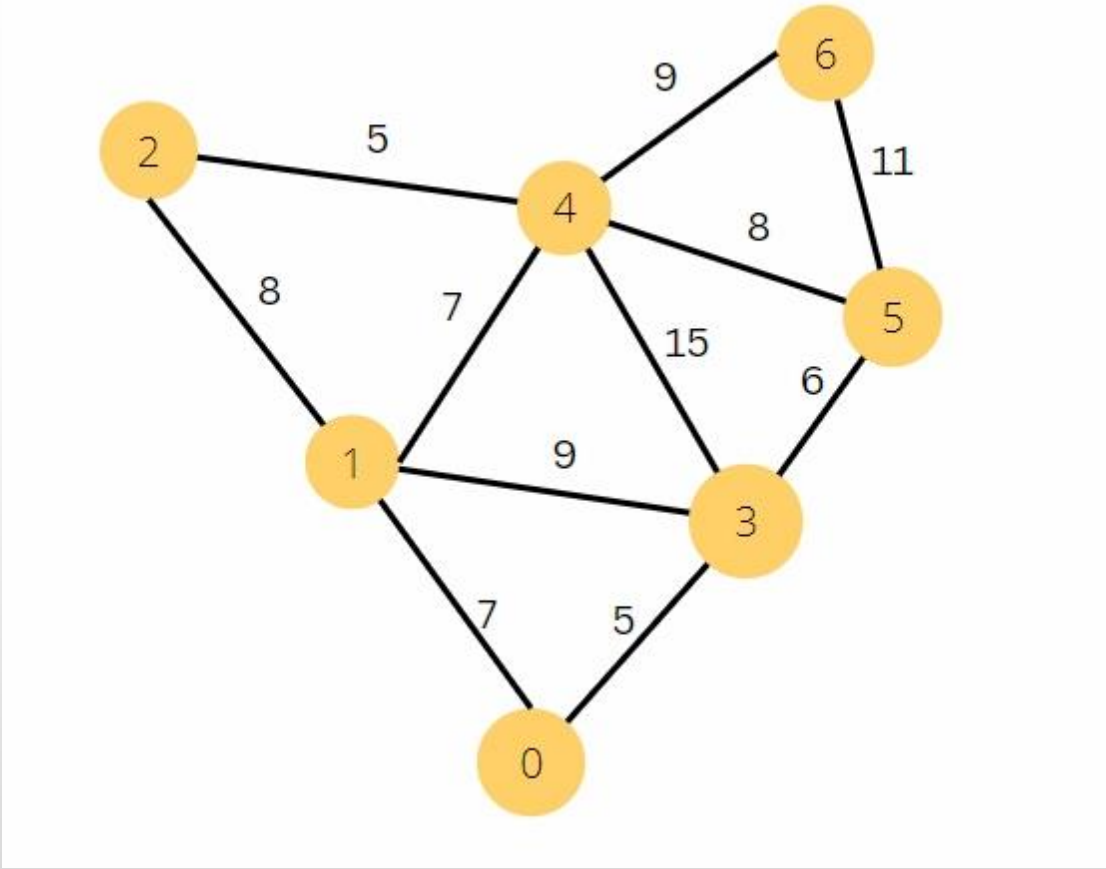
Reverse Delete Algorithm

- Tüm kenarları ağırlığına göre azalan sırayla sırala.
- En yüksek ağırlığa sahip kenarı sil.
- Eğer çizge hala bağlı değilse, silinen kenarı geri ekle.
- Tüm kenarlar silinene kadar 2. ve 3. adımları tekrarla.

Ters Silme Algoritması Özellikleri

- Minimum kapsayan ağacı bulur.
- Greedy (Açgözlü) bir algoritmadır.
- Ağırlık sıralaması yaparak kenarları siler ve çizgenin bağlılığını korur.

Ters Silme



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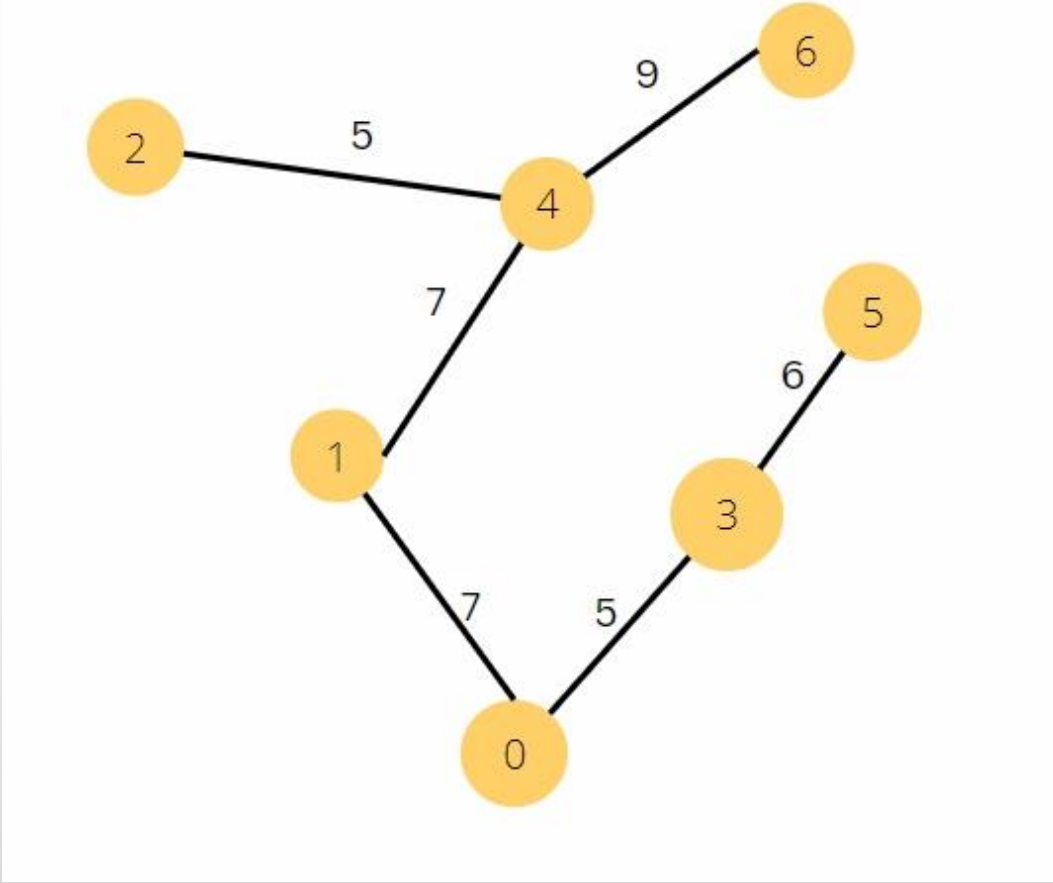

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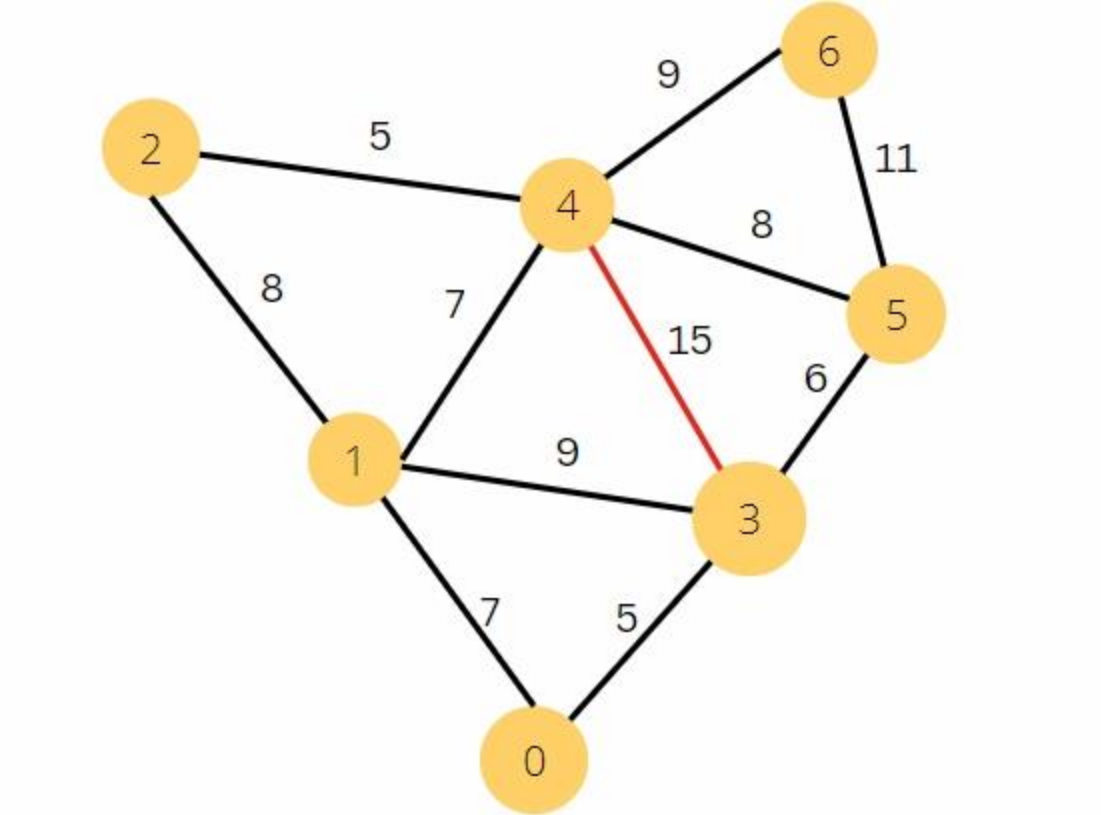

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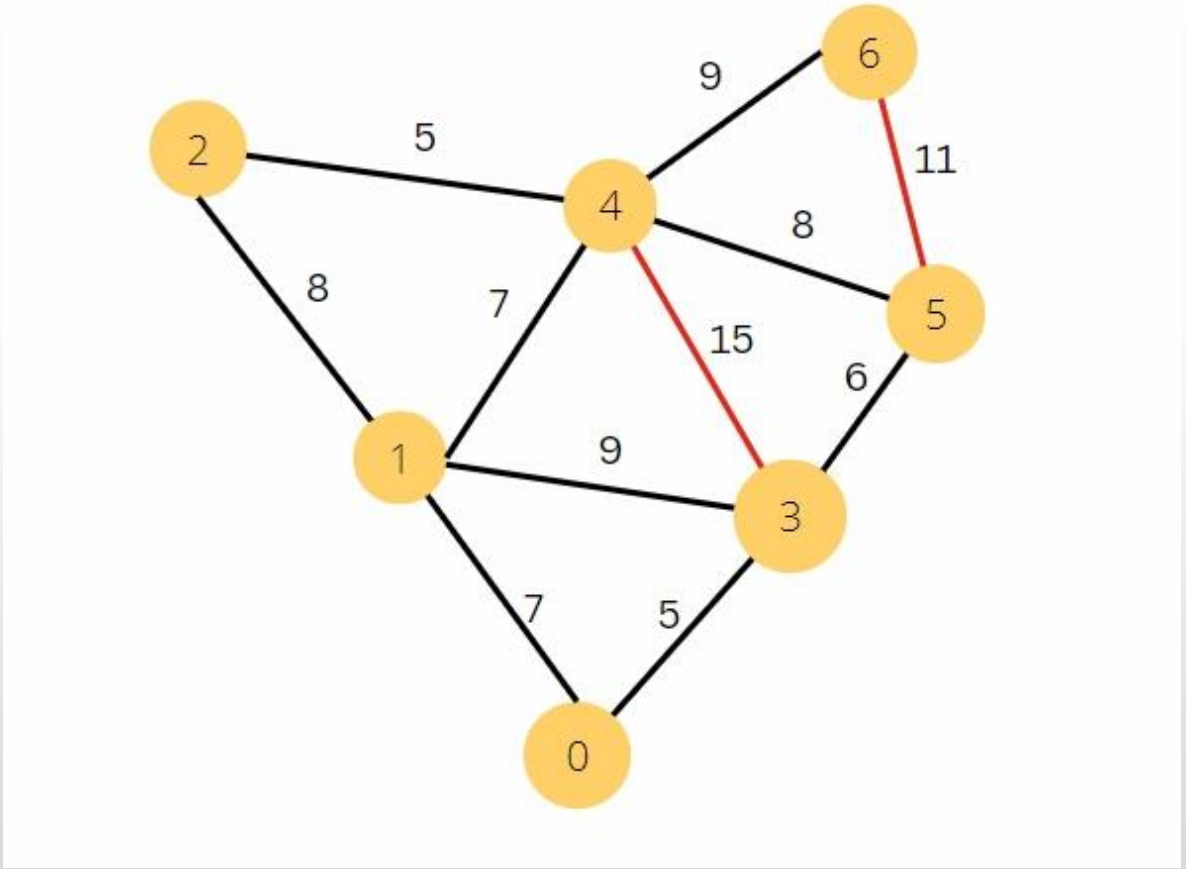

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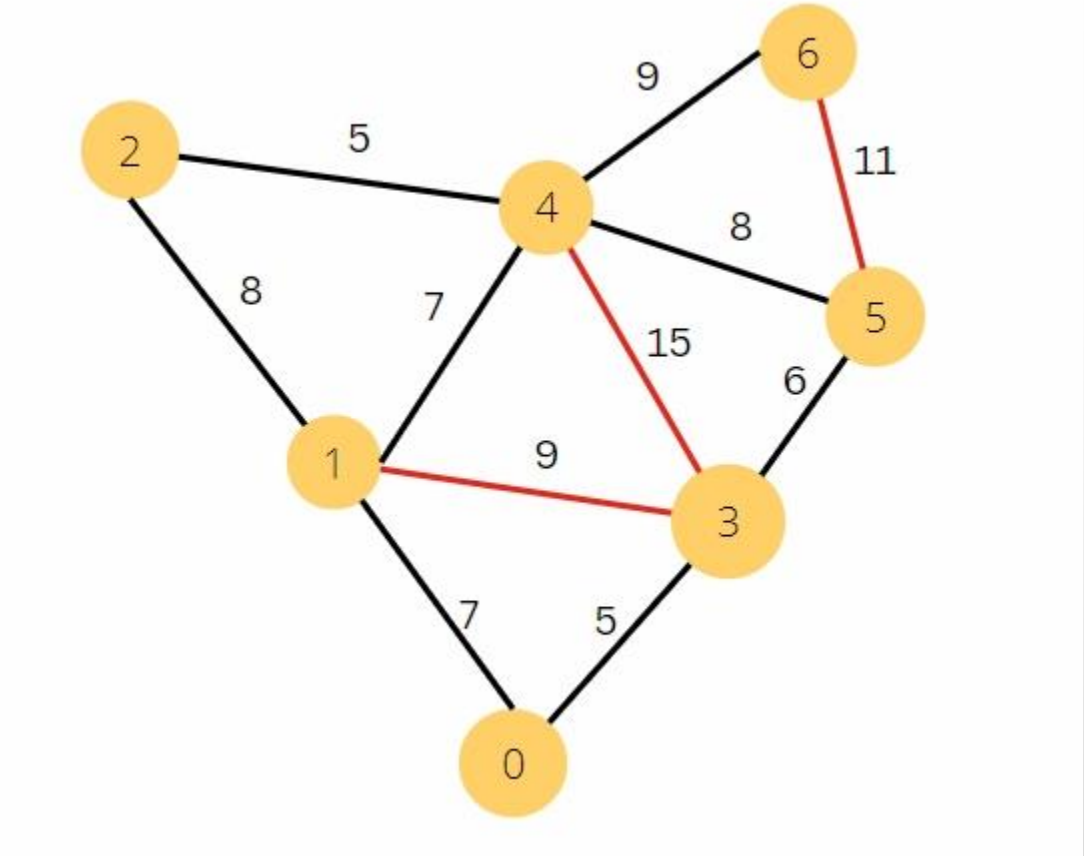

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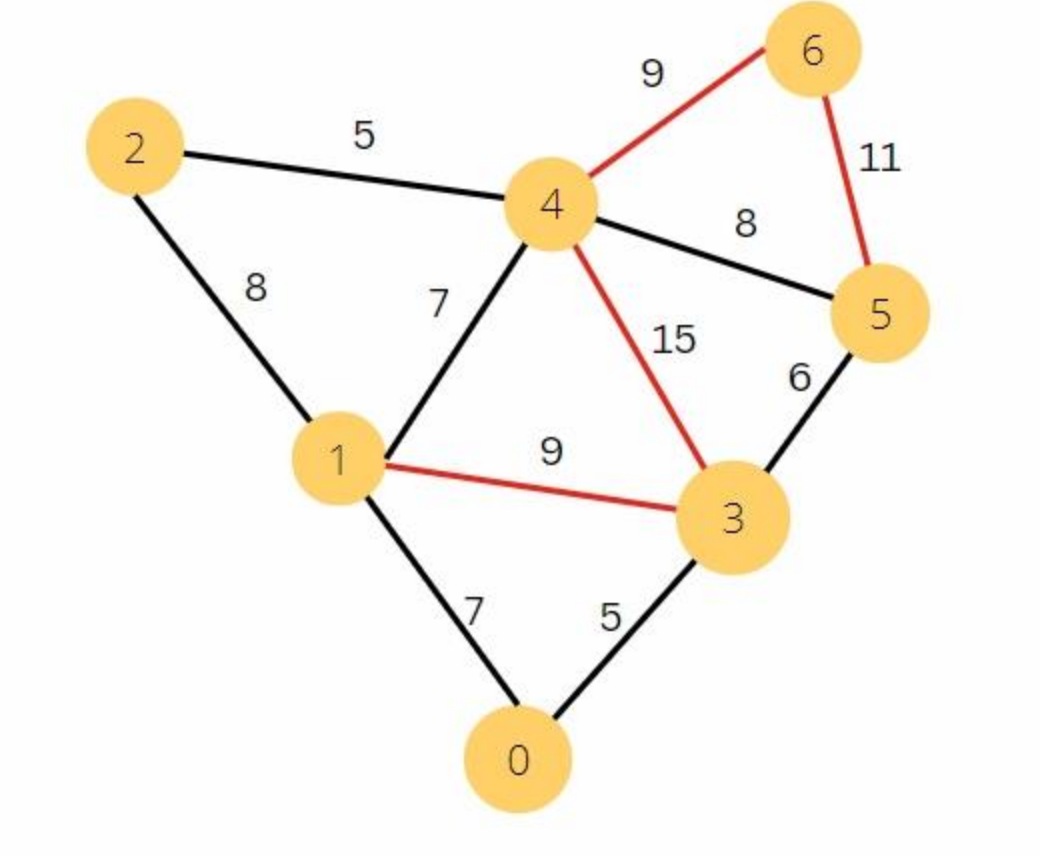

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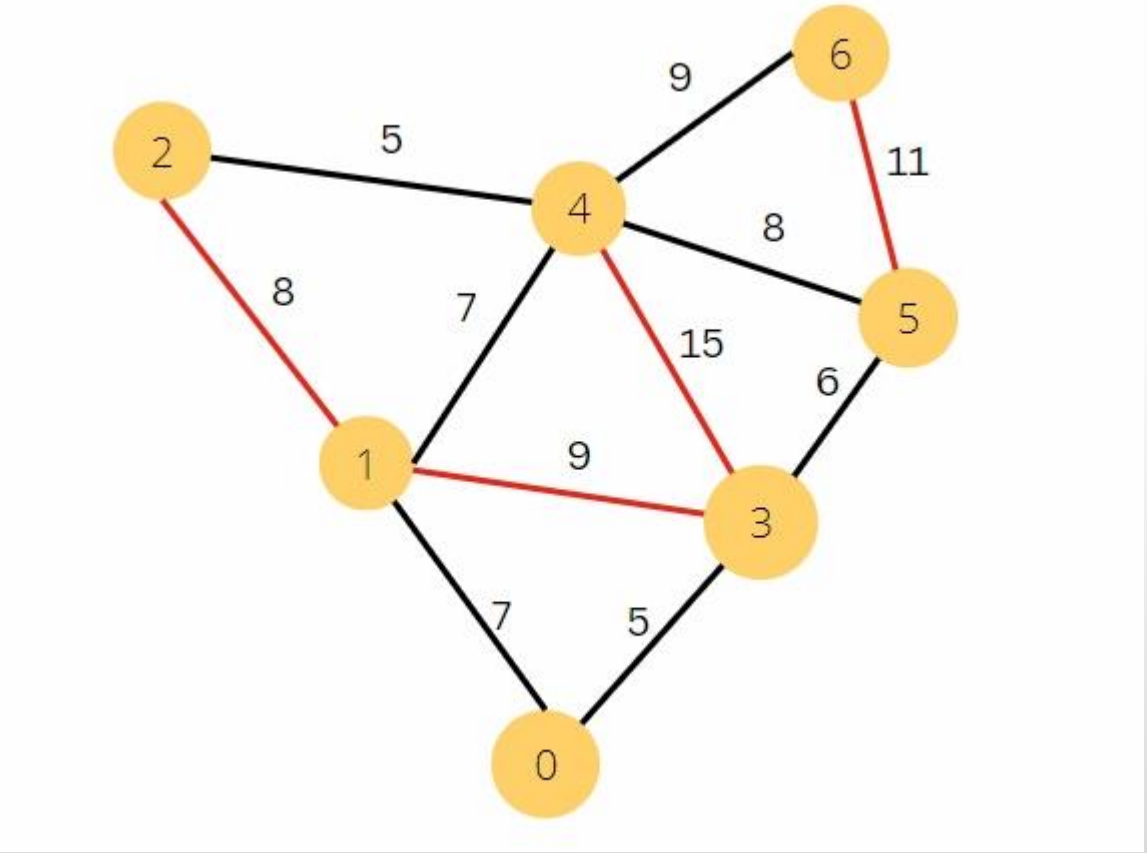

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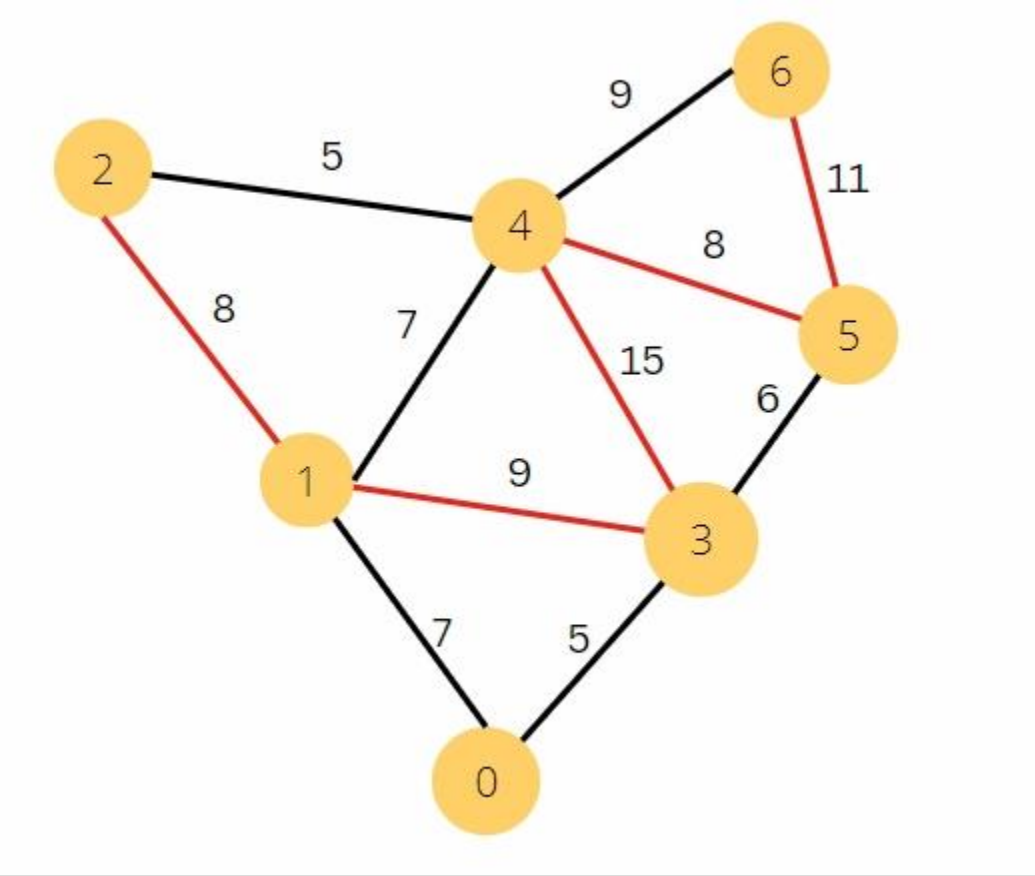
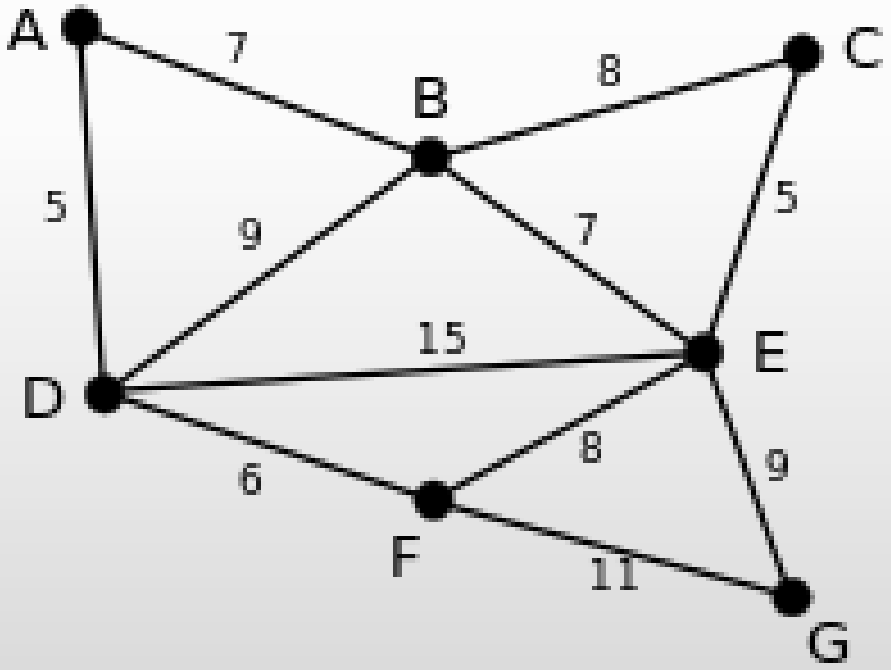


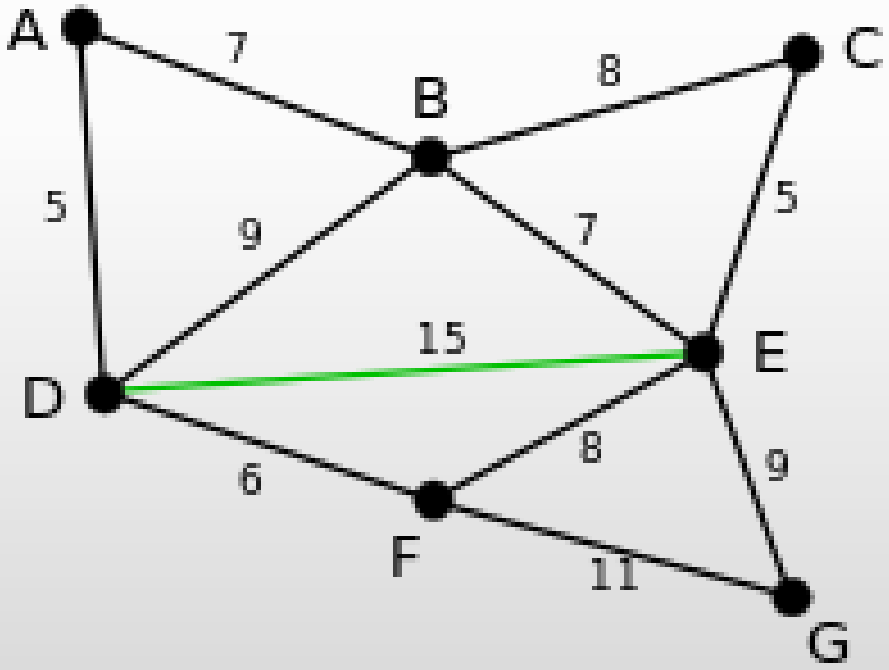
Reverse Delete

- Aşağıdaki çizge verilsin. Sayılar kenarların ağırlıklarını gösterir.



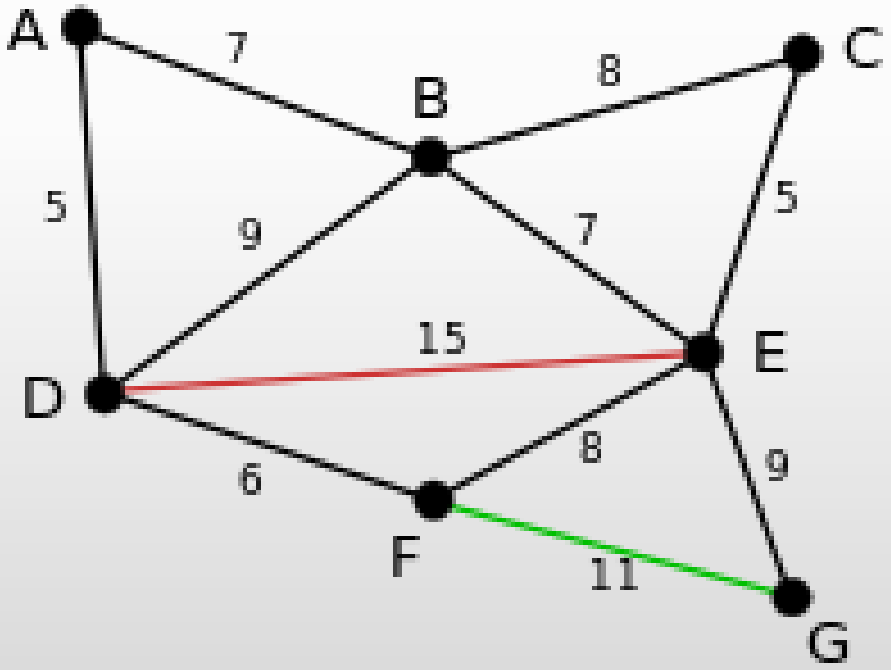
Reverse Delete

- En yüksek ağırlıklı DE kenarı incelenir.



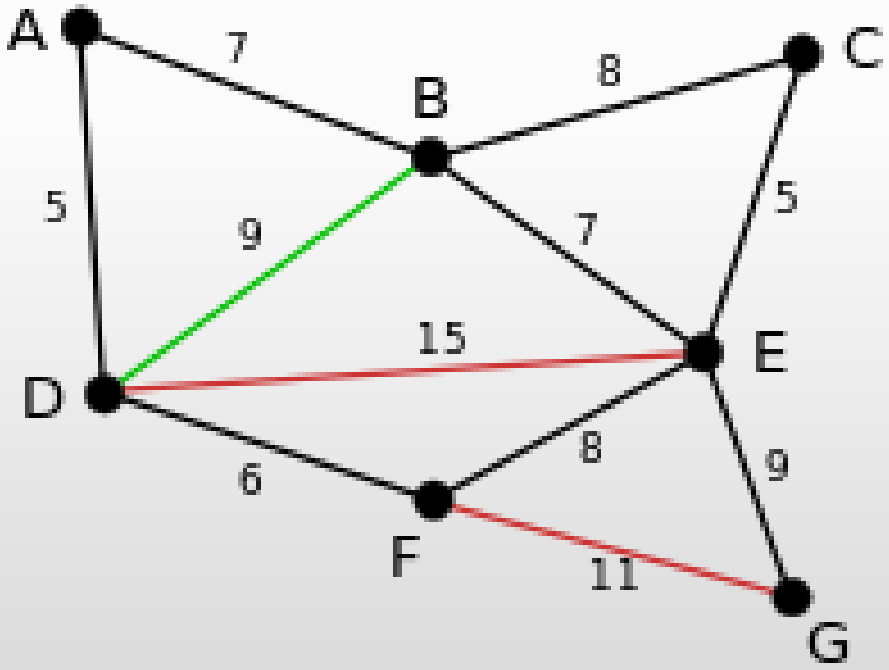
Reverse Delete

- FG kenarı çıkarılırsa çizge bağlılığı bozuluyor mu diye kontrol edilir.



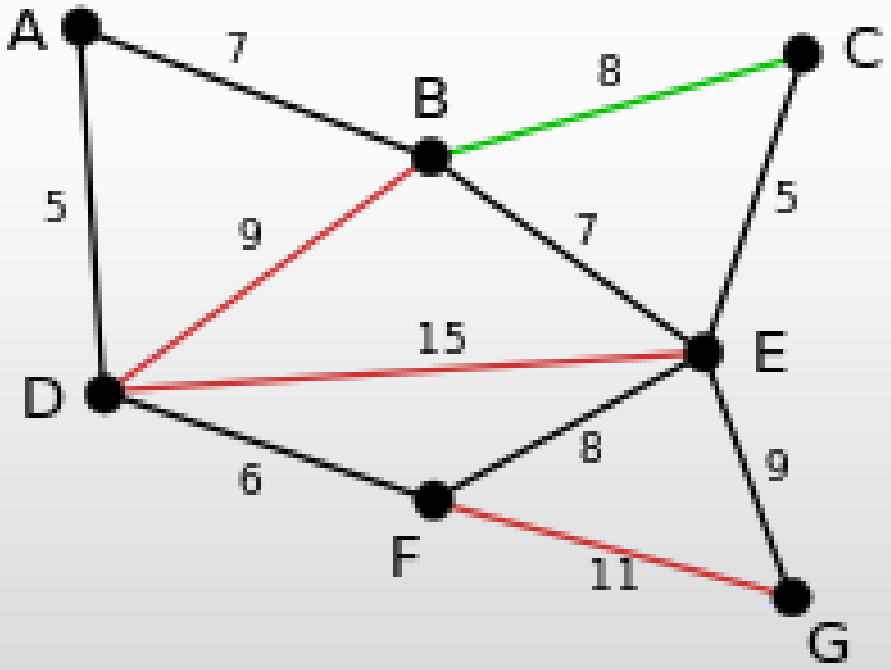
Reverse Delete

- Bir sonraki en ağırlıklı kenar BD kontrol edilir.



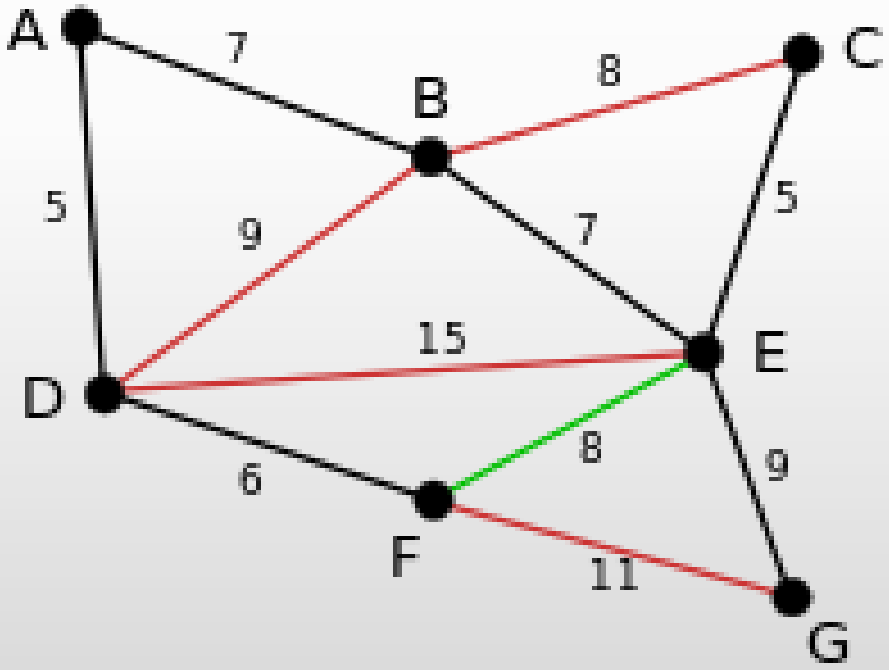
Reverse Delete

- EG kenarı kontrol edilir, silinemeyeceği görülür. BC kenarı incelenir.



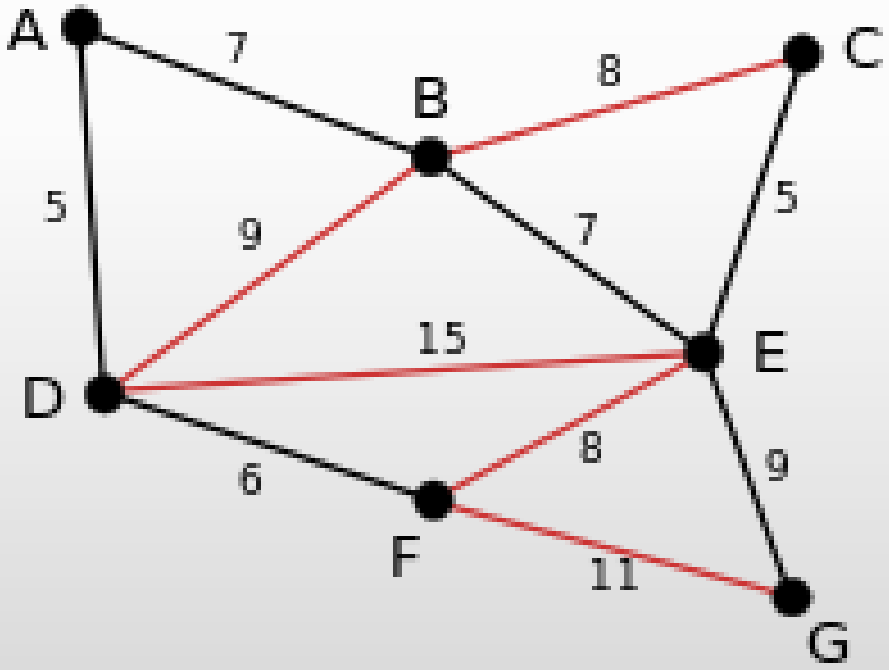
Reverse Delete

- EF kenarı incelenir.



Reverse Delete

- Silinecek başka bir kenar bulunamaz.



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